CAMBRIDGE
INTERNATIONAL EXAMINATIONS

JUNE 2002

GCE Advanced Level
GCE Advanced Subsidiary Level

MARK SCHEME

MAXIMUM MARK : 75

SYLLABUS/COMPONENT :9709 /3, 8719 /3

MATHEMATICS
(Pure 3)

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1 *EITHER*: Express LHS in terms of $\cos\theta$ and $\sin\theta$ or in terms of $\tan\theta$ — M1
Make sufficient relevant use of double-angle formula(e) — M1
Complete proof of the result — A1

OR: Express RHS in terms of $\cos\theta$ and $\sin\theta$ or in terms of $\tan\theta$ — M1
Express RHS as the difference (or sum) of two fractions — M1
Complete proof of the result — A1 — **3**

[SR: an attempt ending with $\dfrac{1-\tan^2\theta}{\tan\theta} = \cot\theta - \tan\theta$ earns M1 B1 only.]

2 *EITHER*: Show correct (unsimplified) version of the x or the x^2 or the x^3 term — M1
Obtain correct first two terms $1 + x$ — A1
Obtain correct quadratic term $2x^2$ — A1
Obtain correct cubic term $\frac{14}{3}x^3$ (allow $\frac{28}{6}$, 4.67, 4.66 for the coefficient) — A1

[The M mark may be implied by correct simplified terms, if no working is shown. It is not earned by unexpanded binomial coefficients involving $-\frac{1}{3}$, e.g. ${}^{-\frac{1}{3}}C_1$ or $\begin{pmatrix}-\frac{1}{3}\\2\end{pmatrix}$.]

[An attempt to divide 1 by the expansion of $(1-3x)^{\frac{1}{3}}$ earns M1 if the expansion has a correct (unsimplified) x, x^2, or x^3 term and if the partial quotient contains a term in x. The remaining A marks are awarded as above.]

OR: Differentiate and calculate f(0), f′(0), where $f'(x) = k(1-3x)^{-\frac{1}{3}-1}$ — M1
Obtain correct first two terms $1 + x$ — A1
Obtain correct quadratic term $2x^2$ — A1
Obtain correct cubic term $\frac{14}{3}x^3$ (allow $\frac{28}{6}$, 4.67, 4.66 for the coefficient) — A1 — **4**

3 Attempt to find a and/or quadratic factor by division or by inspection — M1
Obtain partial quotient or factor $x^2 - x$ — A1
State answer $a = 6$ — B1
State or imply the other factor is $x^2 - x + 3$ — A1 — **4**

[The M1 is earned if division has produced a partial quotient $x^2 + bx$, or if inspection has an unknown factor $x^2 + bx + c$ and has reached an equation in b and/or c.]
[SR: a correct division with unresolved constant remainder can earn M1A1B0A1.]
[NB: successive division by a pair of incorrect linear factors, e.g. $x - 1$ and $x + 2$ or $x + 1$ and $x + 2$, can earn M1A0 or M1A1(if their product is of the form $x^2 + x + k$).]

4	(i) Use the formula correctly at least once	M1	
	State α = 1.26 as final answer	A1	
	Show sufficient iterations to justify α = 1.26 to 2d.p., or show there is a sign change in the interval (1.255, 1.265)	A1	3
	(ii) State any suitable equation in one unknown e.g. $x = \frac{2}{3}\left(x + \frac{1}{x^2}\right)$	B1	
	State exact value of α (or x) is $\sqrt[3]{2}$ or $2^{\frac{1}{3}}$	B1	2
5	Obtain derivative $\pm 2\sin x + k\cos 2x$ or $\pm 2\sin x + k(\cos^2 x \pm \sin^2 x)$	M1	
	Equate derivative to zero and use trig formula to obtain an equation involving only one trig function	M1	
	Obtain a correct equation of this type e.g. $2\sin^2 x + \sin x - 1 = 0$ or $\cos 2x = \cos(\frac{1}{2}\pi - x)$	A1	
	Obtain value $x = \frac{1}{6}\pi$ (allow 0.524 radians or 30°)	A1	
	Show by any method that the corresponding point is a maximum point	A1	
	Obtain second value $x = \frac{5}{6}\pi$ (allow 2.62 radians or 150°), and no others in range	A1✓	
	Determine that it corresponds to a minimum point	A1	7
6	(i) State or imply $f(x) \equiv \frac{A}{(3x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)}$	B1	
	State or obtain $A = -3$	B1	
	State or obtain $B = 2$	B1	
	Use any relevant method to find C	M1	
	Obtain $C = 1$	A1	5
	[Special case: allow the form $\frac{A}{(3x+1)} + \frac{Dx+E}{(x+1)^2}$ and apply the above scheme ($A = -3$, $D = 1$, $E = 3$).]		
	[SR: if f(x) is given an incomplete form of partial fractions, give B1 for a form equivalent to the omission of C, or E, or B in the above, and M1 for finding one coefficient.]		
	(ii) Integrate and obtain terms $-\ln(3x+1) - \frac{2}{(x+1)} + \ln(x+1)$	B1 + B1 + B1✓	
	Use limits correctly	M1	
	Obtain the given answer correctly	A1	5

7	(i) State that $\frac{dm}{dt} = k(50-m)^2$	B1	
	Justify $k = 0.002$	B1	2
	(ii) Separate variables and attempt to integrate $\frac{1}{(50-m)^2}$	M1	
	Obtain $\pm\frac{1}{(50-m)}$ and $0.002t$, or equivalent	A1	
	Evaluate a constant or use limits $t = 0$, $m = 0$	M1	
	Obtain any correct form of solution e.g. $\frac{1}{(50-m)} = 0.002t + \frac{1}{50}$	A1	
	Obtain given answer correctly	A1	5
	(iii) Obtain answer $m = 25$ when $t = 10$	B1	
	Obtain answer $t = 90$ when $m = 45$	B1	2
	(iv) State that m approaches 50	B1	1
8	(i) State or imply a simplified direction vector of l is $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, or equivalent	B1	
	State equation of l is $\mathbf{r} = \mathbf{i} + \mathbf{k} + \lambda(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or $\frac{x-1}{3} = \frac{y}{-1} = \frac{z-1}{2}$, or equivalent	B1✓	
	Substitute in equation of p and solve for λ, or one of x, y, or z	M1	
	Obtain point of intersection $-2\mathbf{i} + \mathbf{j} - \mathbf{k}$	A1	4
	[Any notation is acceptable.]		
	(ii) State or imply a normal vector of p is $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$	B1	
	EITHER: Use scalar product to obtain $a + 3b - 2c = 0$	M1	
	Use points on l to obtain two equations in a, b, c e.g. $a + c = 1$, $4a - b + 3c = 1$	B1✓	
	Solve simultaneous equations, obtaining one unknown	M1	
	Obtain one correct unknown e.g. $a = -\frac{2}{3}$	A1	
	Obtain the other unknowns e.g. $b = \frac{4}{3}$, $c = \frac{5}{3}$	A1	
	OR: Use scalar product to obtain $a + 3b - 2c = 0$	M1	
	Use scalar product to obtain $3a - b + 2c = 0$	B1✓	
	Solve simultaneous equations to obtain one ratio e.g. $a : b$	M1	
	Obtain $a : b : c = 2 : -4 : -5$, or equivalent	A1	
	Obtain $a = -\frac{2}{3}$, $b = \frac{4}{3}$, $c = \frac{5}{3}$	A1	
	[NB: candidates may transfer from the *EITHER* to *OR* scheme by subtracting the two "point" equations, or transfer from *OR* to *EITHER* by finding one of the "point" equations.]		
	OR: Calculate the vector product $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})\times(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$	M1	
	Obtain answer $-4\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}$, or equivalent	A1✓	
	Substitute in $-4x + 8y + 10z = d$ to find d, or equivalent	M1	
	Obtain $d = 6$, or equivalent	A1	
	Obtain $a = -\frac{2}{3}, b = \frac{4}{3}, c = \frac{5}{3}$	A1	
	OR: State or imply a correct equation of the plane e.g. $\mathbf{r} = \lambda(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) + \mathbf{i} + \mathbf{k}$	M1	
	State 3 equations in x, y, z, λ, and μ, e.g. $x = 3\lambda + \mu + 1$, $y = -\lambda + 3\mu$, $z = 2\lambda - 2\mu + 1$	A1✓	
	Eliminate λ and μ	M1	
	Obtain equation $-4x + 8y + 10z = 6$, or equivalent	A1	
	Obtain $a = -\frac{2}{3}, b = \frac{4}{3}, c = \frac{5}{3}$	A1	6
	[SR: condone the use of $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ for $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ in the *EITHER* scheme and the first *OR* scheme.]		

9	(i) State or imply that $r = 2$	B1	
	State or imply that $\theta = \frac{1}{3}\pi$ (allow 1.05 radians or 60°)	B1	
	Obtain modulus 4, and argument $\frac{2}{3}\pi$ of u^2 (allow 2^2; 2.09 or 2.10 radians or 120°)	B1 + B1✓	
	Obtain modulus 8 and argument π of u^3 (allow 2^3; 3.14 or 3.15 radians or 180°)	B1✓	5
	[Follow through on wrong r and θ]		
	[SR: if u^2 and u^3 are only given in polar form, give B1✓ for u^2 and B1✓ for u^3.]		
	(ii) *EITHER*: Deduce that $u^2 - 2u + 4 = 0$ from $u^3 + 8 = 0$		
	OR: Verify that $u^2 - 2u + 4 = 0$ by calculation	B1	
	State that the other root is $1 - i\sqrt{3}$, or equivalent	B1	2
	[NB: stating that the roots are $1 \pm i\sqrt{3}$ is sufficient for both B marks.]		
	(iii) Show both points correctly on an Argand diagram	B1	
	Show the correct relevant circle	B1	
	Show line (segment) correctly	B1	
	Shade the correct region	B1	4
	[SR: allow work on separate diagrams to be eligible for the first three B marks.]		
10	(i) State at any stage that the x-coordinate of A is equal to 1, or that A is the point (1,0)	B1	1
	(ii) State $f'(x) = 2\dfrac{\ln x}{x}$, or equivalent	B1	
	Use product or quotient rule for the next differentiation	M1	
	Obtain $2.\dfrac{1}{x}.\dfrac{1}{x} + 2\ln x.\left(\dfrac{-1}{x^2}\right)$, or any equivalent correct unsimplified form	A1	
	Verify that $f''(e) = 0$	A1	4
	(iii) State or imply area is $\int_1^e (\ln x)^2\, dx$	B1	
	Use $\dfrac{dx}{du} = e^u$, or equivalent, in substituting for x throughout	M1	
	Obtain given answer correctly (allow change of limits to be done mentally)	A1	3
	(iv) Attempt the first integration by parts, going the correct way	M1	
	Obtain $(u^2 - 2u \pm 2)e^u$, or equivalent, after two applications of the rule	A1	
	Obtain exact answer in terms of e, in any correct form, e.g. $(e - 2e + 2e) - 2$, or $e - 2$	A1	3

[The substitution in (iii) may be done in reverse i.e. starting with the u integral and obtaining the x integral. The M1A1 scheme applies, but only an explicit statement will earn the B1.]
[The M1A1A1 in (iv) applies to those working in terms of x and obtaining $x((\ln x)^2 - 2\ln x \pm 2)$, or equivalent.]