MATHEMATICS

Paper 9709/11
Paper 11

Key messages

Candidates should be reminded that an answer unsupported by correct working will not gain credit. This was particularly relevant to questions requiring a definite integral to be found where the use of calculator routines is not acceptable.

General comments

In **Questions 4**, **5(ii)**, **8** and **10(iii)** it was common for candidates not to gain accuracy marks through rounding errors. The rubric on the front cover of the question paper regarding the accuracy of different types of answers should be familiar to all candidates and the requirement of a greater degree of accuracy in intermediate working compared to the final answer should also be appreciated.

Comments on specific questions

Question 1

Stronger responses from candidates obtained and evaluated an expression for the required term rather than finding all the terms and selecting the constant term. The latter process produced more errors in evaluation of powers and combinations.

Question 2

The correct possible values of n were often found by setting the given fist derivative to zero and solving the resulting quadratic equation. A small number of candidates who found the two values identified that the higher of the two was the required solution.

Question 3

Fully correct solutions to this question were often seen. These responses reflected a good knowledge of the straight-line equation and the required algebraic and arithmetic processes. Candidates who did not complete the solution often gained credit for finding the gradient of the curve and/or finding c from substitution of the given coordinates into the curve equation.

Question 4

(i) Candidates who identified the process as a geometric progression usually used the nth term formula to successfully find the first term. Candidates who did not identify that the common ratio was 1.1 were still able to gain method marks in both parts. Some candidates attempted to work back from the last term to the first term and were often unable to reach a correct answer.

(ii) The sum to n terms was used correctly by most of the candidates who had identified the geometric progression correctly in **part (i)**. The use of r^{n-1} rather than r^n was a common error made by candidates. Candidates who correctly used non-rounded answers to **part (i)** in **part (ii)** often gained full credit.

Question 5

(i) The processes leading to the given result; clearing the denominator, use of the trigonometric identity and solving the quadratic in sin*x* were often completed successfully. Stronger responses generally did not include algebraic and arithmetic errors.

(ii) Most candidate responses involved the use of the result given in **part (i)**. The most direct route to the two solutions, finding the value of 2*x* – 20 in the third and fourth quadrants, was rarely seen. Responses displaying other methods often led to only one solution.

Question 6

The equation of the straight line in terms of *m* and/or the gradient of the curve were often found correctly. When the condition for a line to be a tangent to a curve was known candidates were usually able to equate the line and curve equations and set the discriminant of the resulting quadratic to zero to find the two values of *m*, then *x* and then *y*. Candidates who equated the gradient of the line to the gradient of the curve were often less successful and often found difficulty obtaining the quadratic in *x* only.

Question 7

(i) Whilst the upper limit of the range of f was often seen, the existence of a lower limit was not apparent in most responses. More candidates were able to identify the range of g. Alternative notations were accepted, although they had to be correct to gain credit.

(ii) The correct technique for finding a composite was often seen stronger responses demonstrating the removal of the reciprocal of *x* from the denominator to reach the required form**.**

(iii) Finding an inverse by change of subject and swapping the variables was used successfully by most candidates who obtained an answer to **part (ii)**. The final removal of the reciprocal of *x* in the denominator was not commonly seen.

Question 8

(i) The formula for arc length was often quoted and used correctly by most candidates. Some responses did not gain the final accuracy mark as the final answer was not given to the required accuracy or was rounded incorrectly.

(ii) Those candidates who realised *AB* or *CB* could be found using simple trigonometry usually found the required perimeter correctly.

(iii) Calculation of the required triangle areas and the sector area was often seen. Many candidates who found the areas went on to use them correctly to find the shaded area. Candidates who used five-figure accuracy in their intermediate calculations were able to obtain the final answer to the required degree of accuracy.

Question 9

There were many complete attempts to all three parts of this question.

(i) The requirement to integrate the first derivative appeared to be well understood and the use of a constant of integration to complete the method was seen after most of the attempts to integrate.

(ii) The method of differentiation to obtain the second derivative was often completed correctly making this the best answered question part on the paper.

(iii) In most attempts the value of the *x* coordinate was found correctly by setting the first derivative equal to zero. The corresponding value of the *y* coordinate was seen less often and sometimes simply quoted incorrectly as zero.

Although the second derivative was available to be used to identify the turning point some candidates used the change in the sign of the gradient. To gain credit for this method the *x* values used and the sign of the gradient at these values had to be clearly shown.

Question 10

(i) Vectors in the direction of *AB* and *BC* were often found and used correctly to give a scalar product of zero. It was important that the calculation of the scalar product was evident in the working.

(ii) It was sufficient to find *DC* and then demonstrate clearly that *AB* and *DC* were parallel. Showing *DC* and AB were both perpendicular to *BC* also provided a satisfactory route to the required conclusion. This was the most frequently omitted question part.

(iii) Some candidates were able to recall and use the formula for the area of a trapezium whilst other candidates treated the trapezium as a composite of a rectangle and a right-angled triangle. Candidates who were able to find the magnitudes of vectors correctly usually successfully found the area.

Question 11

(i) The correct completed square form was often seen and usually led to a correct expression with x as the subject. Candidates who used more complex procedures for finding the completed square form were rarely completely successful.

(ii) The link between the required result in **part (i)** and the volume of rotation about the y-axis was identified by a minority of candidates. Some candidates were able to square x correctly and integrate the resulting function correctly. The final correct answer was seen very occasionally. Although the 'y-axis' was in bold type several candidates instead found the volume of rotation about the x-axis.

MATHEMATICS

Paper 9709/12
Paper 12

Key messages

It has been noted that some candidates are omitting essential working. Candidates should be reminded to include all necessary detail in their working to ensure that all possible marks can be gained. Insufficient detailed working was noticeable in **Question 10(iii)** in which the process of finding the difference of the two numbers obtained by substituting the limits needed to be shown. The use of calculator functions for supplying the answers is not sufficient.

General comments

The paper was generally well received by candidates, with many high quality and detailed responses seen. The paper gave all candidates the opportunity to show what they had learned and understood, with some questions that provided more challenge for candidates.

Comments on specific questions

Question 1

This question was an accessible start to the paper with many candidates demonstrating a good knowledge of the binomial expansion. Candidates were very often able to write down the relevant terms, form an equation and solve it correctly. Weaker responses sometimes did not include the term in x in their expansion of $(1+\frac{x}{2})^6$ and therefore only had one term to equate to 3.

Question 2

Most candidates demonstrated a very good understanding of the techniques required. Some candidates obtained the wrong gradient or did not use the midpoint.

Question 3

This question appeared to be more challenging for candidates, with many candidates not identifying the need to integrate or wrongly combining integration with the equation of a straight line. Many candidates equated $\frac{dy}{dx}$ to the gradient of the line joining the two points to find k. Some candidates realised the need to find the two unknowns by using both points given, in an integrated expression.

Question 4

In both parts of this question many candidates correctly used the formulae for the length of an arc and the area of a sector, although some did not use 2θ as the angle. Many candidates, however, did not find the correct length of ***AT*** or ***BT***, often not realising that angles OAT and OBT were $90°$. Some premature approximation for the lengths in **part (ii)** meant that some candidates' final answers were not sufficiently accurate. Working to at least 4 significant figures should guarantee that the final answer is accurate to 3 significant figures.

Question 5

Many fully correct solutions to this question were seen with stronger responses proving the given result in **part (i)** using Pythagoras' Theorem and carrying out the required differentiation and analysis in **part (ii)**. Weaker responses often had errors in the differentiation, particularly with the $\frac{1}{3}\pi$ and the brackets. This sometimes resulted in unsuccessful attempts at the product rule or forming a function of a function. Candidates who expanded the brackets and differentiated each term separately were generally more successful. Those who used the second differential to determine the nature of the stationary value usually obtained full marks. Candidates who attempted to consider the sign of $\frac{dy}{dx}$ either side of the stationary point did not often show sufficient working. A significant number of candidates did not respond to **part (ii)**.

Question 6

This question, particularly **parts (a)** and **(b)(ii)** were challenging for candidates. In **part (a)** common errors included attempting to expand $\tan(2x+1)$, working in degrees, and using $\pi -$ rather than $\pi +$ to find the required values. Candidates should be reminded to give answers in radians to 3 significant figures and not to 1 decimal place as they do for angles in degrees. **Part (b)(i)** was generally well answered with most candidates identifying the need to use $sin^2\theta + cos^2\theta = 1$ to obtain the given answer in the required form. Some candidates omitted **part (ii)** or substituted the end points of the domain. Stronger responses successfully considered the maximum and minimum possible values of $cos^2 x$.

Question 7

Many correct responses were seen to this question. In **part (i)** many candidates were able to interpret the diagram and the information given to obtain the correct vectors. **Part (ii)** was occasionally omitted by candidates or insufficient working was given in order to fully validate the candidates' answers. In **part (iii)** the method for using the scalar product to find the required angle was familiar to candidates, although sometimes candidates' working was not sufficiently detailed or was inaccurate due to rounding prematurely.

Question 8

The arithmetic series in **part (a)** of this question proved to be very accessible to candidates, with many candidates gaining full marks. Some candidates thought that it was a geometric series and some other candidates identified that it was arithmetic but found the distance that would have been run on the 22^{nd} day rather than the 21^{st}. **Part (b)** appeared to be more challenging for candidates, although many fully correct answers were seen in each of the parts. Candidates who equated the third term divided by the second to the second divided by the first generally needed less working and were more successful than those who used the formulae for ar and ar^2 . Some candidates rounded $\frac{2}{3}$ to 0.67 and their subsequent answers were insufficiently accurate. Some candidates incorrectly stated that the fourth term would be $x-7$, whilst other candidates tried to find a sum to infinity even though their *r* value was greater than 1. Candidates who were unable to find the value of *x* in **part (i)** often omitted the last 2 parts.

Question 9

This question appeared to be very accessible to candidates, with many candidates gaining full marks. In **part (i)** the most common approach was to equate the 2 functions and use the discriminant equal to 0 although some candidates made errors in obtaining the required rearrangement or made errors in the subsequent working. Another approach was to equate the gradients and many candidates usually followed this method successfully. Weaker responses sometimes equated the gradient of *f(x)* to 0 or to *g(x)* rather than its gradient.

In **part (ii)** candidates frequently started their response correctly but sometimes made errors in their working or gave *x* as two values rather than an inequality. In **part (iii)** some candidates found gf(*x*) and then tried to find the inverse of this, rather than the inverse of g and then the composite. Some candidates did all their working correctly but then rejected the solution that *x* could be 0. In **part (iv)** most candidates completed the square correctly but then often, did not give the least value of *f(x)*, or gave the *x* value instead, or gave the coordinates of the minimum point.

Question 10

Part (i) of this question was attempted by most candidates and many fully correct answers were seen. Weaker responses sometimes appeared unsure what to do with the 1 in both the differential and the integral and forgetting to multiply or divide by 2 was quite common. **Part (ii)** was appeared to be more challenging with some candidates attempting to find the coordinates of *B* before they had found *A*. Other candidates sometimes either used *x* as 0 in their gradient or equated the gradient to 0. **Part (iii)** was frequently omitted by candidates or the incorrect limits were used; using the coordinates of *B* or the *y* intercept of the curve when integrating the curve with respect to *x* was quite common. Some candidates successfully integrated the function with respect to *y* but this was a significantly more demanding option. Very few candidates were able to obtain the whole area including the triangle.

MATHEMATICS

Paper 9709/13
Paper 13

Key messages

Some candidates frequently omitted essential working in places, candidates should be advised that an answer unsupported by correct working will not gain credit. This was identified in **Question 11(ii)** in which the process of integration needed to be shown followed by the process of finding the difference of the two numbers obtained by substituting the limits. The use of calculator routines for supplying the answers is not sufficient.

There is typically a question, such as **Question 4**, which requires candidates to find an angle, or lengths of lines or arcs, or areas of regions by various calculations. It is sometimes the case that calculations appear without it being made clear exactly what the calculation is attempting to find, for example, 'Triangle = ...' is often not sufficient if there is more than one possible triangle. 'Triangle *ACB* = ...' makes it clear and can enable method marks to be awarded.

When the question requests that a particular result is to be shown, it is usually necessary to finish with a statement of the required result. This is particularly important in **Questions 5(i)**, **7(ii)**, **10(i)**, **10(ii)** of this paper. For example, **Question 10(i)** needs to finish with the statement, 'Hence *AXB* is a straight line'. 'Shown' is not sufficient.

General comments

The paper was generally well received by candidates and many very good scripts were seen. Almost all candidates seemed to have sufficient time to finish the paper. Some scripts were particularly difficult to read and look as though some of the answers are written in pencil and then superimposed in ink, which tends to give a very unclear image. Candidates should be strongly advised not to do this.

Comments on specific questions

Question 1

In **part (i)** some candidates did not read the question carefully enough and gave only the term in x^2 rather than the terms as far as the term in x^2. On the whole, the first part was well answered. **Part (ii)** was answered reasonably well but some candidates used only the term from one bracket whilst other candidates did not the square the *p* as well as the *x* when expanding the bracket $\left(px - 2x^2\right)^2$.

Question 2

Most candidates completed the square accurately. A few candidates made a sign error when obtaining the inverse function and other candidates retained the ± of the square root rather than proceeding, in this case, to reject the negative root. A large number of candidates gave the incorrect answer of $x > -2$ for the domain of the inverse function, not recognising the significance of the function *g* being defined for $x > 4$. A variety of incorrect variables, rather than *x*, were seen, such as g^{-1}, $g^{-1}(x)$, *y*.

Question 3

Most candidates differentiated correctly and the majority went on to equate to zero and to obtain the correct values of *x*. However, a substantial number of candidates either did not translate this in terms of *a* and *b*, or applied the values the wrong way round.

Question 4

In **part (i)**, some candidates identified that triangle *OAC* is equilateral and were able to write down the answer immediately. Other candidates wrote $\cos CAO = \frac{r}{2r}$ which also gave the answer very quickly. There were candidates who did not identify that angle *ACB* is a right angle or that $AC = r$ and were not able to obtain the correct answer. **Part (ii)** was generally well answered and those who chose the more straightforward approach of subtracting the area of sector *AOC* from the area of triangle *ABC* made fewer mistakes. Those who chose to use angle *OAC* from **part (i)** in order to find the area of triangle *ABC* tended to be on safer ground than those who chose to find *BC* by using Pythagoras' theorem, since *BC* was sometimes seen as $\sqrt{3}$ instead of $r\sqrt{3}$.

Question 5

In **part (i)** almost all candidates wrote down, in terms of *x*, correct expressions for *S* and *V*, however many candidates did not effectively relate the two expressions. As the answer is given in the question candidates are expected to show individual steps clearly. In this case, for example, depending on the method employed, it might be necessary to show the step that $\left(8x^3\right)^{2/3} = 4x^2$. Several candidates did not demonstrate clear logic; very often candidates would write something like:

$$28x^2 = 7\left(8x^3\right)^{2/3} = 28x^2 .$$

This misses out the vital step $\left(8x^3\right)^{2/3} = 4x^2$ and is simply stating that $28x^2 = 28x^2$. Candidates would be advised to instead start with one side of what is to be shown and finish with the other. For example:

$$\begin{aligned} 7V^{2/3} &= 7\left(8x^3\right)^{2/3} \\ &= 7\left(4x^2\right) \\ &= 28x^2 \\ &= S \end{aligned}$$

In questions like **part (ii)**, where the chain rule is used, it is critical that the letters used in the derivative notation is correct. Where $S = 7V^{2/3}$, the derivative of *S* with respect to *V* is denoted by $\frac{dS}{dV}$. Many candidates were able to differentiate accurately and substitute *V* = 1000 but called the result $\frac{dV}{dS}$ and from this point errors were made in the calculation, usually ending with $\frac{14}{15}$ instead of $\frac{30}{7}$. Those who began by differentiating with respect to *x* created more work for themselves as well as increasing the chance of making an error. The chain rule becomes: $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dS} \times \frac{dS}{dt}$.

Question 6

Part (i) was well answered by most candidates, however sign errors were frequently seen, as were errors in choosing the correct set of inequalities for the answer. In **part (ii)**, some candidates often repeated the work they had done in **part (i)** in order to obtain the required values of *k*. It is sufficient in cases like this to simply use the values of *k* already obtained. Some confused work was seen in this part with candidates substituting *k* values into the equation of the curve instead of, or as well as, the equation of the line. Substituting into the equation of the line leads to $y = 3x - 2$ and $y = -\frac{3}{2}x + 1$ to give the equations of the two tangents and solving simultaneously gives $\left(\frac{3}{2}, 0\right)$, hence meeting on the *x*-axis.

Question 7

Part (i) was very well answered with many candidates gaining full marks. While many candidates scored full marks in **part (ii)** by listing four correct solutions, there were also many candidates who, having reached $\cos^2\theta = 1$, $\frac{1}{3}$, did not include the ± when taking the square root or omitted 2 of the 4 solutions, or did not square root.

Question 8

In **part (i)**, significant numbers of candidates did not appear to read the question carefully enough and incorrectly anticipated that what was required, was to integrate in order to obtain f(x). This was what was required in **part (ii)** and most of these candidates, when they had answered **part (ii)** were able to revisit **part (i)**. What was required in **part (i)** was to set $f'(x) = 0$ (or < 0) and solve. Most candidates were successful in reaching $x < \frac{5}{2}$ but many did not take account of the fact that f was defined for $x > \frac{1}{2}$.
Part (ii) was generally answered well although some candidates did not integrate accurately.

Question 9

The simplest approach in **part (i)** is to set $\frac{5k-6}{3k} = \frac{6k-4}{5k-6}$. Candidates who used this approach were almost always successful. Another approach was to find two expressions for ar^2 which leads to $3k\left(\frac{5k-6}{3k}\right)^2 = 6k-4$, but many candidates using this approach could not effectively manipulate the algebra. A significant number of candidates solved the given quadratic which gained no credit in **part (i)** but they were able to use their k values in **part (ii)**.

Question 10

In **part (i)**, most candidates found *AX* and at least one other vector correctly. At this point, often, candidates either showed that one vector is a multiple of another vector (e.g. *AB* = 3*AX*) or used a scalar product to show that the angle between two vectors is 0° or 180°, as appropriate. Candidates then needed to conclude that *AXB* is a straight line. Full marks were often obtained in **part (ii)** by finding *CX* and using the scalar product to show that *CX* is perpendicular to *AX*. Since the answer is effectively given in the question, candidates needed to show their working and not simply state that *CX.AX* = 0. Few candidates successfully answered **part (iii)**, several omitted this part. A minority of candidates identified the connection with *CX* being perpendicular to *AX*, found in **part (ii)**. Candidates who recognised the connection calculated $\frac{1}{2} \times |AB| \times |CX|$. Many attempted longer methods, with varying degrees of success and often found, for example, an angle of the triangle and then used the formula for area of triangle.

Question 11

Part (i) was answered well overall with most candidates gaining all three marks available. In **part (ii)** each of the functions had to be squared individually before integrating each of these individually. Errors in squaring the functions were common, with many candidates obtaining only two terms instead of three. Some candidates used limits of 1 and 3 for one or both functions, instead of 1 and 2, and 2 and 3 respectively. The process of substitution into the integrated function needed to be shown. The two volumes obtained then needed to be added together; some candidates incorrectly subtracted at this point. There were many candidates that provided good responses to this question by avoiding these common errors.

MATHEMATICS

Paper 9709/21
Paper 21

Key messages

Candidates must ensure that when a question asks for an answer in 'exact form', that their answer is exact. If an exact answer is not required then the appropriate level of accuracy, as stated in the rubric on the front of the examination paper, must be used. Candidates should also ensure that they have answered the questions fully.

General comments

Candidates appeared to have sufficient time to attempt the paper, with sufficient space to answer the questions. It was clear that some candidates were well prepared for the examination.

Comments on specific questions

Question 1

(i) Candidates who squared each side of the given equation, obtaining a quadratic equation, often made good progress and many correct solutions were seen. Candidates who attempted to find linear equations or inequalities tended to yield sign errors, with some candidates giving $x = 0$ erroneously. Once a correct critical value had been obtained, most candidates were able to give the correct inequality as a final result.

(ii) The word 'Hence' indicates that work from the previous part of the question can be made use of. Most candidates used their critical value from **part (i)** to form an equation of the form $\ln n = 4$ or similar. Many correct responses of 54.6 were seen, but many candidates did not identify that this question required an integer answer.

Question 2

Candidates needed to expand out the integrand before any progress could be made. Candidates who obtained and used an expanded-out integrand, many were able to integrate to obtain an acceptable form. When substituting the limits, many candidates appeared to use their calculator to obtain the final result, thus losing the exact value required in the final answer.

Question 3

Most candidates used the quotient rule to obtain an expression for the gradient function. Candidates that used the product rule, were usually equally successful. It was necessary to find the value x at the point where $y = 4$. At this point, some candidates stopped working with exact expressions and appeared to use their calculator, losing accuracy as an exact gradient was required.

Question 4

(i) Most candidates used the factor theorem and obtained a correct value for a. Candidates that attempted algebraic long multiplication were not usually as successful in obtaining the correct value for a.

(ii) Most candidates obtained a correct quadratic factor of $4x^2 + 12x + 9$ from either algebraic long division or by synthetic division. Complete correct factorisation was not commonly seen. Many candidates appeared to use their calculator to solve $4x^2 + 12x + 9 = 0$ and work backwards from the result, often obtaining factors of the form $x + \frac{3}{2}$. A final solution containing factors in this form was not acceptable.

(iii) Many candidates did not identify the importance of the word 'Hence' and did not use their answers to the previous question parts. The space given for the answer, together with the mark allocation and the word 'Hence' should highlight to candidates that a reasonably short piece of work was expected. It was intended that candidates work with the solution $x = 2$, which the majority should have obtained in **part (i)**. Some correct responses were seen, however some candidates did not deal with the square root correctly, whilst others gave incorrect results from attempting to use $x = -\frac{3}{2}$.

Question 5

(i) Most candidates demonstrated their knowledge of the rules of integration in answering this question. The given answer was obtained by many candidates who, having originally obtained an error in a coefficient or sign, returned to their original work and corrected it.

(ii) Many candidates did not use $\alpha - \sqrt[3]{3 - 2\sin 2\alpha - \cos\alpha}$ or equivalent, using $\sqrt[3]{3 - 2\sin 2\alpha - \cos\alpha}$ instead. When a correct expression was being used, some candidates did not obtain the expected sign change as their calculators were in the incorrect mode. As the question initially involved calculus, then any angles are deemed to be in radians. Several candidates gave no response to this question.

(iii) Many candidates had their calculators in the incorrect mode for answering this question. Candidates using starting points outside the range $0.5 \leqslant \alpha \leqslant 0.75$ took a long time to converge to the correct result. Many candidates did not give their iterations or their final answer to the required level of accuracy. A common error was that candidates did not write down the last iteration needed to justify their final answer.

Question 6

(a) Most candidates recognised that they could write the initial equation as $\frac{1}{\sin\alpha\cos\alpha} = 7$. Some candidates then used the double angle formula as was intended. Other candidates squared both sides of the equation and used trigonometric identities to obtain a quadratic equation in terms of one trigonometric ratio. Often sign errors or errors in the solution of the linear trigonometric equations obtained, meant that many final solutions were incorrect. Several candidates only gave one solution.

(b) Most candidates were able to simplify the left hand side of the equation to $2\sin\beta\cos 20°$. Many candidates recognised the need to use the tangent ratio, but often evaluated the angles too early together or prematurely approximated which led to an inaccurate solution.

Question 7

(i) Many candidates appeared to manipulate their incorrect work to obtain the given answer. Some candidates did not identify the need for implicit differentiation and were unable to gain many marks. Sign errors, errors in the implicit differentiation of a product and not differentiating 1, were common errors. Some completely correct solutions were also seen. Some candidates did not answer the question as required by omitting to find an explicit expression for $\frac{dy}{dx}$ before making any substitutions.

(ii) Many candidates did not attempt this part of the question. Those that did attempt it often obtained the result $y = \frac{1}{2}x$ but were unable to progress. There were very few complete and correct solutions.

(iii) Many candidates did not complete this part of the question. Those that did attempt it were sometimes able to deduce that $y = x$ but were unable to progress much further than this.

MATHEMATICS

Paper 9709/22
Paper 22

Key messages

Candidates must ensure that when a question asks for an answer in 'exact form', that their answer is exact. If an exact answer is not required then the appropriate level of accuracy, as stated in the rubric on the front of the examination paper, must be used. Candidates should also ensure that they have answered the questions fully.

General comments

Candidates appeared to have sufficient time to attempt the paper, with sufficient space to answer the questions. It was clear that some candidates were well prepared for the examination.

Comments on specific questions

Question 1

Most candidates attempted algebraic long division with varying amounts of success. Many candidates were able to obtain a partial quotient of $x^2 - 3x$, however errors in the simplification of the terms within the algebraic long division meant that some were unable to obtain the fully correct quotient and remainder.

Question 2

(i) Many correct solutions were obtained with the majority of candidates using the method of squaring each side of the given equation to obtain a 3-term quadratic equation. There were some errors in simplification which to an incorrect quadratic equation and also errors in the solution of the quadratic equation either by factorisation or by use of the formula. Candidates who attempted to find two linear equations very often made sign errors when attempting to deal with the modulus.

(ii) Many correct solutions were seen, with most candidates using their positive answer from **part (i)**.

Question 3

It was essential for candidates to recognise that they needed to write the equation of the straight line in the form $\ln y = \ln k + a\ln x$, recognising that the gradient of the line was equal to a and then making a suitable substitution to find $\ln k$ and hence k. Correct substitution of the given points into the equation $\ln y = \ln k + a\ln x$ to form two simultaneous equations was also acceptable, as was solving simultaneously the equations $e^{3.96} = k \times 0.22^a$ and $e^{2.43} = k \times 1.32^a$. Many candidates were unable to progress much further than finding the gradient of the line. Incorrect use of the coordinates given was a common error in this question. Some solutions showed a correct method to obtain the value of a, but followed with an incorrect substitution to attempt to find the value of $\ln k$ and hence k. Premature approximation in earlier calculations in some cases meant that an inaccurate final value for k was obtained. However, there were candidates who produced completely correct, well set out solutions, showing a good understanding of the straight line theory necessary.

Question 4

(i) Most candidates were able to apply the iterative method required and obtain sufficient iterations to the correct accuracy and hence the final answer to the correct accuracy. Common errors included not giving enough iterations to justify the final answer and not giving the final answer to the correct accuracy required. Some candidates misread the question and used the iterative formula $x_{n+1} = \dfrac{1}{\ln(2x_n)}$ rather than the correct $x_{n+1} = \dfrac{x_n}{\ln(2x_n)}$. Candidates must ensure that they read the question carefully.

(ii) Very few correct solutions were seen with many candidates using their answer to **part (i)** to answer this part. The word 'exact' was used in the question, which should have highlighted this error as their answer to **part (i)** was not exact. Some candidates stated a correct equation but did not identify the word 'exact' and gave a decimal equivalent of $\dfrac{e}{2}$.

Question 5

This question was intended that candidates make use of the product rule, equate their result to zero and solve to find x and then y. Use of the quotient rule was also acceptable, so long as a correct rewriting of the exponential term had taken place. Many candidates attempted to use the product rule but were unable to differentiate $e^{-\frac{1}{2}x}$ correctly. When candidates attempted to equate their $\dfrac{dy}{dx}$ to zero, several candidates were unable to do this correctly, not realising that $e^{-\frac{1}{2}x} \neq 0$ and thus attempting to deal with the resulting linear factor in x. There were candidates who, having obtained a correct value for x, did not give an exact value for y.

Question 6

(a) With a given answer to work towards, most candidates attempted to work with a multiple of $\ln x$, or equivalent, from integration. There were many correct integrals of $\dfrac{3}{2}\ln x$ or equivalent. The question demanded that a given result be shown. It was therefore essential that each step of the application of the limits, use of the subtraction rule for logarithms and use of the power rule for logarithms (in either order) be shown. Many candidates did not show the necessary detail needed in a question of this type. Several candidates also used their calculators to attempt to show their decimal result was equal to $\ln 27$.

(b) Candidates needed to identify that a double angle formula was needed to rewrite the integrand. Candidates who made use of the correct double angle formula, often gave an exact answer as required. Several candidates did not successfully deal with integrands involving the square of a trigonometric expression.

Question 7

(i) Many candidates identified the correct approach in this question. There were errors in the coefficient obtained from the parametric differentiation of the given equations and also errors in simplifying these results when applied to $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \times \dfrac{d\theta}{dx}$. Errors also occurred in the simplification of this result when the substitution of $\theta = \dfrac{\pi}{6}$ was attempted.

(ii) Many candidates carried forward their errors in simplification from **part (i)** into **part (ii)**, which made it difficult to progress. Some correct solutions were seen, with the occasional use of degrees rather than radians used in the final answer. Candidates should always ensure that they are working with the correct measure of angles when attempting questions of this type.

Question 8

(i) Most candidates were able to gain marks in this part. It should be noted that the value of the angle was required to 2 decimal places, meaning that several candidates did not gain the final accuracy mark as an answer of $67.4°$ was given. Some candidates made sign errors when attempting to find the value of α. It was also expected for the value of *R* to be evaluated either as 1.3 or an equivalent fraction. A result of $\sqrt{1.69}$ was deemed to be insufficiently simplified.

(ii) Provided the result from **part (i)** was used correctly, many candidates were able to obtain at least one of the solutions. Many candidates had difficulty in dealing with the fact that if $\theta + 67.38° = 52.02°$ a negative angle was obtained and that they should be considering $\theta + 67.38° = 52.02° + 360°$. It was important that candidates appreciate the meaning of the word 'Hence' and use their results from **part (i)**. Several candidates also stated the incorrect $\cos(\theta + 67.38°) = \cos\theta + \cos 67.38°$.

(iii) Very few correct responses were seen to this part. It was intended that the expression from **part (i)** be used so that the given expression could be written as $\left(3 - 2(1.3)\cos(\theta + 67.38°)\right)^2$. Some candidates progressed this far, but did not deduce that they could make use of the maximum value of $\cos(\theta + 67.38°)$ being 1 and that the minimum value of $\cos(\theta + 67.38°)$ is -1. A simple substitution then yields the correct results. Some candidates expanded out the brackets and not progress much further. Candidates should be guided by the mark allocation and in this case, the amount of space given as to the amount of work that is necessary to complete the question.

MATHEMATICS

Paper 9709/23
Paper 23

Key messages

Candidates must ensure that when a question asks for an answer in 'exact form', that their answer is exact. If an exact answer is not required then the appropriate level of accuracy, as stated in the rubric on the front of the examination paper, must be used. Candidates should also ensure that they have answered the questions fully.

General comments

Candidates appeared to have sufficient time to attempt the paper, with sufficient space to answer the questions. It was clear that some candidates were well prepared for the examination.

Comments on specific questions

Question 1

(i) Candidates who squared each side of the given equation, obtaining a quadratic equation, often made good progress and many correct solutions were seen. Candidates who attempted to find linear equations or inequalities tended to yield sign errors, with some candidates giving $x = 0$ erroneously. Once a correct critical value had been obtained, most candidates were able to give the correct inequality as a final result.

(ii) The word 'Hence' indicates that work from the previous part of the question can be made use of. Most candidates used their critical value from **part (i)** to form an equation of the form $\ln n = 4$ or similar. Many correct responses of 54.6 were seen, but many candidates did not identify that this question required an integer answer.

Question 2

Candidates needed to expand out the integrand before any progress could be made. Candidates who obtained and used an expanded-out integrand, many were able to integrate to obtain an acceptable form. When substituting the limits, many candidates appeared to use their calculator to obtain the final result, thus losing the exact value required in the final answer.

Question 3

Most candidates used the quotient rule to obtain an expression for the gradient function. Candidates that used the product rule, were usually equally successful. It was necessary to find the value *x* at the point where $y = 4$. At this point, some candidates stopped working with exact expressions and appeared to use their calculator, losing accuracy as an exact gradient was required.

Question 4

(i) Most candidates used the factor theorem and obtained a correct value for a. Candidates that attempted algebraic long multiplication were not usually as successful in obtaining the correct value for a.

(ii) Most candidates obtained a correct quadratic factor of $4x^2 + 12x + 9$ from either algebraic long division or by synthetic division. Complete correct factorisation was not commonly seen. Many candidates appeared to use their calculator to solve $4x^2 + 12x + 9 = 0$ and work backwards from the result, often obtaining factors of the form $x + \frac{3}{2}$. A final solution containing factors in this form was not acceptable.

(iii) Many candidates did not identify the importance of the word 'Hence' and did not use their answers to the previous question parts. The space given for the answer, together with the mark allocation and the word 'Hence' should highlight to candidates that a reasonably short piece of work was expected. It was intended that candidates work with the solution $x = 2$, which the majority should have obtained in **part (i)**. Some correct responses were seen, however some candidates did not deal with the square root correctly, whilst others gave incorrect results from attempting to use $x = -\frac{3}{2}$.

Question 5

(i) Most candidates demonstrated their knowledge of the rules of integration in answering this question. The given answer was obtained by many candidates who, having originally obtained an error in a coefficient or sign, returned to their original work and corrected it.

(ii) Many candidates did not use $\alpha - \sqrt[3]{3 - 2\sin 2\alpha - \cos\alpha}$ or equivalent, using $\sqrt[3]{3 - 2\sin 2\alpha - \cos\alpha}$ instead. When a correct expression was being used, some candidates did not obtain the expected sign change as their calculators were in the incorrect mode. As the question initially involved calculus, then any angles are deemed to be in radians. Several candidates gave no response to this question.

(iii) Many candidates had their calculators in the incorrect mode for answering this question. Candidates using starting points outside the range $0.5 \leqslant \alpha \leqslant 0.75$ took a long time to converge to the correct result. Many candidates did not give their iterations or their final answer to the required level of accuracy. A common error was that candidates did not write down the last iteration needed to justify their final answer.

Question 6

(a) Most candidates recognised that they could write the initial equation as $\frac{1}{\sin\alpha\cos\alpha} = 7$. Some candidates then used the double angle formula as was intended. Other candidates squared both sides of the equation and used trigonometric identities to obtain a quadratic equation in terms of one trigonometric ratio. Often sign errors or errors in the solution of the linear trigonometric equations obtained, meant that many final solutions were incorrect. Several candidates only gave one solution.

(b) Most candidates were able to simplify the left hand side of the equation to $2\sin\beta\cos 20°$. Many candidates recognised the need to use the tangent ratio, but often evaluated the angles too early together or prematurely approximated which led to an inaccurate solution.

Question 7

(i) Many candidates appeared to manipulate their incorrect work to obtain the given answer. Some candidates did not identify the need for implicit differentiation and were unable to gain many marks. Sign errors, errors in the implicit differentiation of a product and not differentiating 1, were common errors. Some completely correct solutions were also seen. Some candidates did not answer the question as required by omitting to find an explicit expression for $\frac{dy}{dx}$ before making any substitutions.

(ii) Many candidates did not attempt this part of the question. Those that did attempt it often obtained the result $y = \frac{1}{2}x$ but were unable to progress. There were very few complete and correct solutions.

(iii) Many candidates did not complete this part of the question. Those that did attempt it were sometimes able to deduce that $y = x$ but were unable to progress much further than this.

MATHEMATICS

Paper 9709/31
Paper 31

Key messages

Candidates must ensure that they show clear and detailed working, particularly when a question asks to obtain a given answer. Candidates must ensure they write clearly and do not overwrite one solution with another as this can often make their response illegible. Candidates must ensure that when a question asks for an answer in 'exact form', that their answer is exact. If an exact answer is not required then the appropriate level of accuracy, as stated in the rubric on the front of the examination paper, must be used.

General comments

Most candidates showed a good understanding of the topics covered. In many instances, a candidate made a correct start to a question but were not able to continue this to obtain the correct answer. For example, in integrating $\cos 3x$ in **Question 10** they obtained $\sin 3x$ or 3sin3x. Errors in basic algebra and arithmetic often limited the marks that candidates gained.

Most candidates showed confidence in dealing with the inequality in **Question 2**, and the partial fractions in **Question 8**. Candidates also showed a good basic knowledge of calculus in **Question 3**, **Question 6** and **Question 8(ii)**. The questions that candidates found most challenging were the differential equation in **Question 4** and the complex numbers in **Question 10**.

Comments on specific questions

Question 1

Most candidates made the correct first step or rewrote the given equation as $1+e^{2y}=e^{2x}$. The next step of rearranging to make *y* the subject of the equation proved more challenging for candidates. The majority of candidates provided an answer to the question but errors in applying the laws of logarithms were often seen.

Question 2

Candidates who started by squaring both sides of the inequality usually obtained the correct critical values, provided that they squared the 4 as well as squaring the two brackets. Some candidates considered a pair of linear equations or inequalities, however candidates who followed this route often made errors in their arithmetic. Candidates who drew a sketch to accompany their work were often more confident in reaching the correct conclusion. Several fully correct solutions were seen.

Question 3

Many candidates understood the method for obtaining an expression for $\frac{dy}{dx}$. Several errors were noted in the coefficients at the first attempt, but the given answer enabled stronger candidates to identify and correct any errors. Candidates who mistakenly thought that $\frac{dy}{dt}=\frac{1}{1-\cos 2t}$ did not often correct themselves. Some candidates obtained a factor of $\left(1-\cos^2 2t\right)$ in the denominator but did not progress any further. As candidates were working towards a given answer they were expected to give a fully correct solution.

Question 4

(i) Few candidates identified the need to begin with the constant of proportionality and deduce its value. Most candidates started with the given equation and showed that the numbers fitted, which does not sufficiently answer the question.

(ii) Most candidates gained the first two marks in this question for correct separation of variables and obtaining the term $\ln N$. Several errors in the coefficient when integrating the exponential term were noted, but most candidates then successfully used the boundary conditions to evaluate their constant of integration. Several processing errors were noted in the final stage of the solution, and only a minority of candidates reached the correct final answer. Some candidates attempted to use the boundary conditions before doing any integration.

(iii) Very few correct answers were seen. The majority of answers claimed, 'it gets bigger' or 'it gets smaller'.

Question 5

A significant proportion of candidates did not attempt to answer this question, or part of it.

(i) Some candidates made little progress here. Most candidates identified from the inclusion of the phrase 'stationary point' that they needed to start with differentiation. Many of these candidates recognised the function as a product and applied the appropriate rule correctly. Some candidates made errors in dealing with the 2 in e^{-2x}, but several candidates did obtain the given result correctly.

(ii) To answer this question candidates needed to distinguish between solving $x = f(x)$ and solving $f(x) = 0$. Candidates need to calculate values of an appropriate function and then complete their solution with an appropriate comment. Several candidates calculated values for $x = f(x)$ but then made errors in their answers to try to achieve a sign change. The two functions most commonly seen using $f(x) = 0$ were $f(x) = \ln(x-1) - \frac{1}{2(x-1)}$ and $f(x) = -2e^{-2x}\ln(x-1) + \frac{e^{-2x}}{x-1}$.

(iii) Several candidates did not provide a response to this part. Candidates who offered a solution often showed good knowledge and understanding of the process. Although working to more than 4 decimal places was accepted for the individual iterations, the final answer needed to be given correct to 2 decimal places. Some candidates gave their final answer to 2 significant figures rather than to 2 decimal places.

Question 6

(i) Most candidates appeared to be familiar with this result and how to prove it. Many fully correct solutions were seen.

(ii) Several candidates did not provide a response to this part. The first key step was for the candidates to recognise the need to use integration by parts, and then to use the result given in **part (i)**. Although many candidates claimed to have reached the given solution, only a few candidates demonstrated this with fully correct method.

Question 7

(i) The majority of candidates understood the process for finding the point of intersection of two lines. They expressed the position vector in component form and set about finding and solving appropriate linear equations. Common errors were often due to slips with signs when forming equations or in the arithmetic.

(ii) Several candidates did not provide a response to this part. Amongst those who attempted the question, all three of the alternatives shown on the mark scheme were seen. The most common method was use of the vector product to find a vector normal to the plane. The most common errors were sign errors and arithmetic errors in completing the vector product.

Question 8

(i) Most candidates successfully answered this question. Those who selected an appropriate form for the partial fractions usually reached a correct answer. The most common error, apart from in arithmetic, was to omit either the term with denominator x, or the term with denominator x^2.

(ii) Most candidates used their fractions for the integration. The basic forms of the integrals were often recognised, but several errors in the coefficients were noted. As the final answer was given, candidates needed to show full and clear working to demonstrate how they had reached the given result.

Question 9

(i) Most candidates were familiar with the process for demonstrating the given result in this question, and several fully correct solutions were seen.

(ii) Although the question says 'hence', several candidates were unable to make any progress with simplifying the given equation to an equation they could solve. Candidates who reached the correct cubic equation sometimes appeared to be uncertain about the need to find the cube root of a negative number, and several attempted use of the inverse trigonometric function before cube-rooting.

(iii) Candidates who answered this question correctly often used the result from part **(i)**. The alternative of using $\cos^3 x = \cos x\left(1-\sin^2 x\right)$ was also seen. Few fully correct solutions were seen, since there were often errors in coefficients within the integration.

Question 10

A significant proportion of candidates did not attempt to answer this question, particularly part **(ii)**.

(i) Some candidates appeared to be confused between the square and the square roots. Candidates who attempted the correct method for finding the square roots often obtained a correct pair of simultaneous equations in a and b. The process for solving those equations was not often recognised. Some candidates obtained a correct quadratic in a^2 or b^2, but the correct final answer was rarely seen.

(ii) Some candidates attempted to draw a circle of radius 3, but the centre was often in the wrong quadrant. The interpretation of $\arg z$ was often correct, but when a candidate uses different scales on the two axes this is not always clear. The line for $\operatorname{Im} z = 2$ was sometimes confused with the line for $\operatorname{Re} z = 2$.

MATHEMATICS

Paper 9709/32
Paper 32

Key messages

Candidates must ensure that they show clear and detailed working, particularly when a question asks to obtain a given answer. Candidates must ensure they write clearly and do not overwrite one solution with another as this can often make their response illegible. Candidates must ensure that when a question asks for an answer in 'exact form', that their answer is exact. If an exact answer is not required then the appropriate level of accuracy, as stated in the rubric on the front of the examination paper, must be used.

General comments

The paper appeared fairly challenging for many candidates with **Questions 1** on logarithms and indices, **Question 3** on factors and remainders, **Question 5** implicit differentiation and **Question 7** on complex numbers, were not well answered by many candidates.

It was noted that several candidates did initial work in pencil and then over-wrote in pen, this can result in work being illegible. It is important to remind candidates not to do this, and that it is preferable to put a line through work and re-write it if corrections are necessary.

A number of simple errors in basic algebra or arithmetic were seen in candidates' responses throughout the paper. Attention to detail is an essential part of good mathematical work. Among the errors seen, not using brackets appropriately is the most common, by omitting these, this can often result in incorrect further work and loss of accuracy.

Candidates commonly did not give answers to the required degree of accuracy or in the required form and often did not include sufficient detail of working which was required.

Comments on specific questions

Question 1

Most candidates presented confident responses to this question, however, many started with the incorrect statement $5\ln\left(4-3^x\right)=5\ln 4-5\ln 3^x$. A large proportion of the candidates who started with the correct statement $4-3^x=e^{1.2}$ made no further progress because they then continued to take logarithms term by term afterwards. There were several errors noted in calculator work; the correct expression $\frac{\ln\left(4-e^{1.2}\right)}{\ln 3}$ did not always lead to a correct answer. Some candidates appeared to show lack of confidence in manipulating logarithms, which was reflected in the commonly seen, incorrect step $\frac{\ln A}{\ln B}=\frac{A}{B}$. Several candidates did however identify that using logarithms in base 3 was an efficient way to finish.

Question 2

The majority of candidates started with a correct application of the quotient rule. There were some errors in misquoting the rule, and some candidates omitted the denominator altogether. Candidates who made their working clear by setting out their *u* and *v* and quoting the correct rule usually scored the first M mark. A few candidates used the product rule, but the derivative of $(1-x^2)^{-1}$ proved challenging.

Most candidates recognised the need to equate the derivative to zero, but subsequent errors in algebraic manipulation led to many a sign error and an incorrect quadratic. The majority of candidates did reject the inadmissible solution at the end.

Question 3

This question appeared to be particularly challenging for candidates. Most candidates started by attempting to divide x^4+3x^3+ax+b by x^2+x+1. The first step of this division was often correct, but subsequent errors in the algebra and arithmetic, including errors caused by not allowing for the absence of an x^2 term in the quartic, led to many incorrect quotients and remainders. Some difficulties also arose when dealing with the signs of terms involving *a* and *b* : $b+1$ in place of $b-1$ in the remainder was common, despite having everything correct up to that point. Dealing with variations of $(a+2)x+x$ in the remainder also caused some confusion with x appearing and disappearing. Having reached a linear remainder, many candidates did not go on to set their remainder identically equal to $2x+3$, and to solve for *a* and *b* accordingly.

Some candidates opted to use the remainder theorem, but the majority of these used decimal values for the roots of x^2+x+1 rather than work through with exact surd values. Other candidates substituted the value for the root in the quartic, but not in the remainder.

A few candidates opted for the variation of subtracting $2x+3$ from $p(x)$ before doing the long division and setting their remainder equal to 0. A common error was for candidates to treat $2x+3$ as a factor.

Question 4

(i) There were many fully correct solutions to this part of the question. Almost all candidates obtained $R=\sqrt{7}$ and the majority used a correct method to find α, although many gave this to only 1 or 2 decimal places. The most successful students expanded using the compound angle formula and showed full working. Other candidates recited a formula for carrying out the procedure, although effective in some cases, the error $\tan\alpha=\sqrt{6}$ was common. Some candidates started with the incorrect statements $\sin\alpha=1$ and $\cos\alpha=\sqrt{6}$.

(ii) Most candidates found this question challenging. Many candidates did not work with enough accuracy in answering this question, given that **part (i)** required an answer to 3 decimal places, candidates should have identified that at least 3 decimal places was needed for this part. A common incorrect answer was 13.5. Some candidates attempted to acknowledge the connection between the two parts but solved for 2θ rather than for θ or tried to solve $\sqrt{7}\sin(2\theta+44.416)=2$.

A few candidates disregarded their work from **part (i)** and used alternative methods, usually involving double-angle identities, however these rarely resulted in a correct solvable equation in a single trigonometric function.

Question 5

The majority of candidates started by using implicit differentiation to differentiate the left hand side of the equation. A lot of correct work was seen, but also several sign errors due to omission of brackets when differentiating the second term were noted. Very few candidates did not obtain zero as a result of differentiating the right hand side; several continued with a^3, and a few had $3a^2$.

The phrase 'the tangent is parallel to the *x*-axis' was not understood by all candidates; it was often interpreted as $x=0$ or $y=0$ or the denominator of the expression for the derivative is zero or, occasionally, $\frac{dy}{dx}=1$. Those candidates who correctly set the derivative equal to zero usually obtained $y=4x$ but they often overlooked the possibility of $y=0$. Very few candidates earned the mark for mentioning and rejecting $y=0$, instead a few selected this and rejected $y=4x$. Several candidates did not progress further than $y=4x$. Those who did refer back to the original equation to find *y* often made errors in dealing with the cubes and the error $(4x)^2=4x^2$ was common.

Question 6

The majority of candidates recognised that the correct first step was to separate the variables, and most gained the first two marks by getting as far as $\ln(x+2)$. Most candidates went on to obtain $k\ln\left(\sin\frac{1}{2}\theta\right)$ but only a minority had $k=2$. Many candidates found difficulty in the correct simplification and removal of the logarithms. Several candidates obtained a correct expression in terms of $\sin\frac{1}{2}\theta$ or $\cos\frac{1}{2}\theta$, but did not progress further than this. Very few fully correct solutions were seen as the majority of candidates did not attempt the final step to express the answer in terms of $\cos\theta$.

Question 7

(a) Many candidates found this question challenging, often candidates would confuse their working involving *z* or *z** rather than *x* and *y*. Candidates who started by multiplying up by *z** or *x* – i*y* usually reached a correct horizontal equation. Those who tried dealing with the $\frac{iz}{z^*}$ by turning it into $\frac{iz^2}{z^*z}$ often made arithmetic and algebra errors in their working. Only a minority of candidates who reached a horizontal equation in *x* and *y* went on to consider the real and imaginary parts.

(b) (i) Several candidates did not provide a response to this question part. Of those who responded, several candidates demonstrated an understanding of the equations of basic loci. Some candidates did not draw a recognisable circle, however several circles with the correct radius and centre were seen. Many candidates found drawing the line $\operatorname{Im} z = 3$ challenging, with several candidates drawing a second circle.

(ii) Many of the stronger responses identified *P* and obtained the correct answer quite easily, with some adding relevant details to their diagram in **part (i)** by way of explanation. Many candidates used their inaccurate diagrams from **part (i)**, with several candidates tried taking measurements from their diagrams, so 1.7, and 1.8 often appeared in the working.

Question 8

(i) This question was well received by the majority of candidates. Most candidates set up an appropriate form of partial fractions, though a minority started with $\frac{B}{x^2+2}$ in place of $\frac{Bx+C}{x^2+2}$. In this question many candidates did not include brackets, which was a common error seen. There were also several cases of miscopying $2x-1$ as $2x+1$. After fully correct working, several candidates misstated their conclusion as $\frac{4}{2x-1}-\frac{1}{x^2+2}$, this however did not affect the marks gained in this part.

(ii) Most candidates integrated their $\frac{k}{2x-1}$ to get a term in $\ln(2x-1)$, but an error in the multiplying constant was common. For candidates who found $C=0$ in **part (i)** the second term was often integrated correctly, although it was common to see an erroneous extra *x* in the integral. Several candidates started from $\frac{-1}{x^2+2}$ instead of $\frac{-x}{x^2+2}$ and gained no credit for an apparently 'correct' integral as it came from wrong working. When *C* was found to be non-zero, candidates then needed to split this term into 2 parts.

Those candidates who had the correct forms of both integrals usually substituted limits correctly, but some made errors in combining logarithms and some candidates did not gain the final mark by not putting the answer into the required format.

Question 9

(i) The majority of candidates attempted to use integration by parts. Most candidates clearly understood the basic process, but there were many errors in the coefficients. It was common to see $\frac{1}{3}$ and $\frac{1}{9}$ in place of 3 and 9, and several candidates did not include the 3 at all. Those who did complete the integration correctly usually went on to use the limits correctly and reach the given result.

(ii) Many candidates appeared to be unsure of whether they were required to look for solutions of $\mathrm{f}(a) = a$ or look for solutions of $\mathrm{f}(a) = 0$. Very few of those candidates who evaluated $\dfrac{4 - 3\cos\frac{a}{3}}{\sin\frac{a}{3}}$ at 2.5 and 3, went on to make the relevant comparisons with 2.5 and 3; many candidates seemingly adjusted their answers to demonstrate a sign change.

Several candidates evaluated an expression such as $\mathrm{f}(a) = a\sin\frac{1}{3}a + 3\cos\frac{1}{3}a - 4$ for a = 2.5 and a =3 and they often completed the argument correctly. Some candidates simply stated '> 0' and '< 0' without giving values, this was not sufficient detail to gain the mark.

Some candidates appeared to be working in degrees, rather than radians as stated in the question stem. There were also some candidates that did not use an appropriate formula, in some cases they often gained or omitted a 3.

(iii) Some candidates appeared to be working in degrees in answering this question, however many candidates answered this question well. Some candidates did not work to the accuracy requested in the question, and some did not give a final conclusion even though they worked through the iteration correctly.

Question 10

(i) The majority of candidates showed a good understanding of the method required, with errors in signs and algebra being a common occurrence. Several candidates started with a correct expression in component form, but then set the scalar product with $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ equal to zero rather than 5. A small number of candidates tried to remove the fractions in the final answer and multiplied their position vector by 3, thus not gaining the final mark.

(ii) The majority of candidates used the correct method to find the angle between the line and the vector perpendicular to the plane. However, many did not give an acute angle as their final answer, and many did not give the angle between the line and the plane. Answers such as $50°$, $130°$ and $-40°$ were common.

(iii) Correct application of the vector product resulted in concise, correct solutions from many candidates. This was by far the most popular and successful method, although sign errors were seen in some solutions. Candidates who used the alternative method of stating two scalar product equations tended to find it relatively difficult to complete the method. Common incorrect methods usually involved confusion between directions and position vectors.

MATHEMATICS

Paper 9709/33
Paper 33

Key messages

Candidates need to ensure that they think about the best approach to adopt when solving a problem as seen in **Question 1** and **Question 2**. It is important for candidates, where a question asks for a specific method to be used, to use this method, as seen in **Question 10(ii)**. In questions that have a given answer, **Question 4(i)**, it is essential for candidates present all detailed working within their response.

General comments

The questions that candidates found more accessible were **1**, **4(i)**, **4(ii)**, **5(ii)**, **5(iii)** and **9(i)** whilst the questions candidates found more challenging were **5(i), 6(iii)**, **7(ii)**, **7(iii)**, **8(ii)**, **9(ii)** and **10(ii)**.

Candidates should be reminded to present their work clearly and logically, by working systematically down the page.

Often candidates included sufficient details in their working, but candidates should be reminded that they must think carefully about the most appropriate method for solving a particular problem.

Comments on specific questions

Question 1

Many fully correct answers were seen. Some candidates did not present their final answer in a suitable form. While most candidates solved a quadratic equation, a more efficient method was to consider a pair of linear inequalities, leading immediately to the correct domain.

Question 2

The most efficient approach was to apply the remainder theorem twice. Candidates doing so usually gained full marks, although a few candidates incorrectly used the remainder + 24 instead of – 24. Many candidates used long division; this often led to errors in at least one of the equations as the algebra became more challenging, with two unknown constants.

Question 3

Some candidates were able to obtain a correct equation from which they could reach the answer by taking logs, showing the steps in their working. Candidates often did not reach the correct equation, for example because they took the log of each individual term after cross-multiplying, or they changed 4.3^{2x} into 12^{2x}, or 4.3^{-x} into $\frac{1}{(4.3^{x})}$, etc. Many algebraic errors were seen throughout the responses to this question.

Question 4

(i) Most candidates scored 3 or 4 marks on this question part. There were some candidates who gained only one mark for the expansion of tan(3x) as they did not form an equation by including the 3cot$x$ term.

(ii) Many candidates were successful in answering this question. A common error was to find inverse tan directly from t^2 instead of first obtaining t.

Question 5

(i) Candidates appeared to find this question quite challenging. Often candidates' graphs did not cover the whole domain for which the functions are defined, so $\ln(x + 2)$ needed $x > -2$ and $4e^{-x}$ needed $-\infty < x < \infty$. Most candidates restricted their sketches to $x > 0$. The sketch of $\ln(x + 2)$ should have shown a clear asymptote at $x = -2$ and a positive decreasing gradient, but not with y = constant as an asymptote. The sketch of $4e^{-x}$ should have passed through (0, 4) rather than (0, 1), with the positive x-axis as an asymptote and going to $+\infty$ as x tended to $-\infty$.

(ii) Nearly all candidates successfully evaluated an appropriate function at $x = 1$ and at $x = 1.5$ then made a correct argument concerning different signs.

(iii) Almost all candidates gained full marks on this part.

Question 6

(i) The evaluation of x and y from $x^2 + y^2 = 1$ and $\frac{y}{x} = \sqrt{\frac{3}{2}}$ was regularly seen. This approach usually led candidates to two solutions so they needed to reject one of them. $w = 1 + \frac{i\pi}{3}$ was a common incorrect response to this question part. The answer could have been written down immediately from $x = r\cos(\arg)$ and $y = r\sin(\arg)$.

(ii) The position of u given in the Argand diagram was nearly always correct, but sometimes with a missing scale. For Argand diagrams it is important that the x and y scales are identical. Many candidates incorrectly highlighted that the imaginary coordinate of v was 4i when instead this point should have been shown to be at $r = 2\sqrt{5}$ and $\arg = \tan^{-1}(2) + \frac{\pi}{3}$.

(iii) Given the modulus and argument of v, it was necessary for candidates to establish that the modulus of $2uw$ was the same: $|2uw| = 2.|u|.|w| = 2.|u|.1 = 2|u|$, and $\arg(2uw)$ was the same: $\arg(2uw) = \arg(2) + \arg(u) + \arg(w) = \arg(u) + \frac{\pi}{3}$. Errors in arithmetic and algebra were often identified with uw being multiplied correctly but terms in the denominator then being multiplied by 2.

Question 7

(i) Most candidates successfully found the value of d, however many candidates had the coordinates of point P for the values of the normal a, b and c.

(ii) Many candidates used the correct formula but often with $+ d$ instead of $- d$. It was very common to see the denominator with point P used instead of the normal to the plane. Other candidates omitted the square root sign from the denominator.

(iii) Few candidates knew how to obtain the direction of the line by taking the cross product of the normal to the plane with $\overrightarrow{OP}$. Many candidates who did obtain the correct direction vector often did not gain the final mark as the vector equation of the line should include '**r =**' rather than the scalar 'l ='

Question 8

(i) Many candidates found this trapezium rule question challenging. The question clearly asked for an estimate for the area under the curve between $x = 0$ and $x = 1.2$, however a significant number of candidates used the upper limit $\frac{\pi}{2}$ instead, whilst other candidates used the wrong number of intervals. Sometimes when two intervals were used, some candidates expressed the values incorrectly in degrees while for others it was difficult to see what candidates had been used for sec(0), sec(0.6) and sec(1.2). In order to obtain the final answer correct to 2 d.p. it was necessary to work to a greater degree of accuracy throughout.

(ii) The question requested candidates to use the diagram and the trapezium rule. Most candidates made a statement about the gradient of the curve, however by sketching the two trapezia on the graph this would have helped them to explain that the area was greater than that under the curve.

(iii) Candidates who opted to differentiate using the quotient rule made good progress with this section, however a considerable number of candidates did not differentiate 1 correctly and hence made errors. Other candidates who used the chain rule often missed out a sine term and hence made little progress.

Question 9

(i) There were some good responses to this question part. The separation of variables was usually correct, as were partial fractions and integration with respect to *t*. Integration of $\frac{A}{(20-x)}$ and $\frac{B}{(40-x)}$ terms was also generally correct. The most common error occurred after finding the value of the arbitrary constant when candidates attempted to exponentiate. Starting with $a + b$ they should have obtained e^{a+b} rather than $e^a + e^b$.

(ii) This part required the use of their answer to **part (i)**, therefore a fully correct response was not often seen.

Question 10

(i) In order to make progress in this part it was necessary to apply the product rule correctly once and then the chain rule correctly twice. Several candidates had the incorrect sign in the product rule and most other candidates had at least one chain rule error, for example a derivative of $e^{\cos x}$ as $e^{\cos x}$ or $\cos x e^{\cos x}$ and the derivative of $\sin^3 x$ as $3\sin^2 x$.

(ii) Many candidates did not do the requested substitution and attempted integration by parts instead; candidates who used this method often made little progress. Candidates who did attempt to use the given substitution often reached the correct expression in *u*, i.e. $(u^2 - 1)\,e^u$. However, candidates often changed their limits incorrectly, with an upper limit $u = 1$ instead of $u = -1$. If this working was followed through without further errors this would finish with the negative value of the correct answer, which was often seen with a changed the sign for no apparent reason to achieve a positive answer. The integration of $(u^2 - 1)\,e^u$ by parts (applied twice) would be better done as a continuous piece of working rather than on different parts of the page as this often led to sign errors. Most candidates omitted brackets, leading to sign errors. It was very common in the integration by parts to see e^u integrated to $\frac{e^u}{u}$.

MATHEMATICS

Paper 9709/41
Paper 41

Key messages

Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Cases where this was commonly not adhered to were seen in **Question 2**, **Question 3**, and **Question 5**. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.

When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation or a work/energy equation. Such a diagram would have been particularly useful for **Question 3**, **Question 4** and **Question 6**.

In questions such as **Question 6**, where velocity is given as a cubic function of time, it is important to identify that calculus must be used and that it is not possible to apply the equations of constant acceleration.

General comments

The paper was generally very well answered by many candidates, although a wide range of marks was seen.

Most candidates work was well presented, but candidates must be reminded of writing their answers clearly in either a black or dark blue pen.

In **Question 4**, the angle α was given exactly as $\tan\alpha = \frac{7}{24}$. In cases such as this, candidates should be reminded that they do not need to evaluate the angle, as this type of problem can often lead to exact answers and so any approximation of the angle can lead to a loss of accuracy.

The examination allowed candidates at all levels to show their knowledge of the subject, whilst still differentiating well between the stronger candidates. **Questions 1, 2** and **3** were found to be the most accessible questions whilst **Questions 4** and **7** proved to be the most challenging.

Comments on specific questions

Question 1

This question requires use of the equation $P = Fv$ relating the power, P, the driving force F and the velocity v. In this case the driving force must exactly equal the weight of the load since the system is moving with constant speed and therefore zero acceleration. The driving force must be $1250g$ and by applying the equation $P = Fv$ with the given power of 20000 W, the speed V can be found. Almost all candidates used this approach successfully.

Question 2

In this question the resistance force is not constant since the work done against resistance is given as 2000 J and so the work-energy principle must be used here. As the speeds are given at both the bottom and top of the hill, the loss in kinetic energy can be found. The distance travelled up the hill and the angle of the slope are both given and so it is possible to evaluate the gain in potential energy. A simple application of the work–energy principle in the form WD by F + KE loss = PE gain + WD against resistance enables the driving force F to be found. Some candidates used 0.7km, rather than 700 m, this is particularly important since there must be consistency with units used. In some case the signs of terms in the work energy equation were also incorrect.

Question 3

(i) In this question it is first necessary to draw a force diagram. It can then be seen that the normal reaction R is given by $R = 3g\cos 60$. As the block is on the point of moving the equation $F = \mu R$ can be used for friction. A simple application of the equilibrium of forces parallel to the plane leads to a 3-term equation involving the weight component, friction and the applied 15 N force. Rearranging this will give the value of μ.

(ii) In this question the greatest value of X will be when the block is about to move up the plane. This means that the equation of equilibrium parallel to the plane shows that X is equal to the sum of the weight component down the plane and the friction force which also acts down the plane. Most candidates identified this and used the value of μ which was found in the first part of the question.

Question 4

(i) This question is an example of a system of connected particles. The two blocks must be modelled as particles. It would be useful for candidate to draw a force diagram before starting, showing clearly which forces are acting on A and B. There are three possible equations of motion here. Newton's second law could be applied either to A or to B or to the combined system of A and B. Any two of these equations would enable the problem to be solved. It must be remembered that the tension in the string is pulling particle B down the slope whereas the same tension is pulling particle A up the slope. By considering the whole system, the two tension forces are not involved, and this will lead directly to an equation for the acceleration of the particles. Use of one of the other equations will give the tension in the string. Some candidates did not include all of the relevant forces and sometimes lost accuracy by evaluating the angle when it was not necessary.

(ii) This part is a simple application of the constant acceleration equations. The acceleration was found in **part (i)** and so the problem uses $a = 1.2$, $u = 1$ and $s = 0.65$. There are several approaches that could be taken such as using the equation $s = ut + \frac{1}{2}at^2$ which leads to a quadratic equation in t. Most candidates made a good attempt at this part, using their value of a from 4**(i)**.

Question 5

(i) In this question the most straightforward method, which was used by most candidates, was to resolve forces horizontally and vertically. This produced two simultaneous equations involving F and θ in the form of $R\sin\theta$ and $R\cos\theta$. One of several possible methods of solution is to use trigonometry such as $\tan\theta = \frac{R\sin\theta}{R\cos\theta}$ and this will enable θ to be found and then this value can be used in one of the equations found from resolving forces to find the value of R. Most candidates made a good attempt at this question; however, some candidates prematurely approximated the answers during their working. Other candidates confused the components by interchanging the sine and cosine terms. Overall this question was well attempted by most candidates.

(ii) This part of the question requires use of Newton's second law along the rod AB. The given forces must be resolved along the rod and then this force is equated to mass x acceleration. The most common seen was when candidates considered the forces both parallel to the rod and perpendicular to it and then using Pythagoras to find the total force. However, the ring is constrained to move only along the rod and hence the need for the force in that direction only.

Question 6

(i) In this question there are two possible approaches, either use of conservation of energy or use of constant acceleration equations. In the energy approach the potential energy lost can be determined by using the given height of 1.8 m as PE lost = $1.8mg$ and this can be equated to the gain in kinetic energy, KE gain = $\frac{1}{2}mv^2$ which enables the speed of the particle as it enters the water to be found. Similarly, the constant acceleration equations would use $a = g$, $s = 1.8$, $u = 0$ and application of the equation $v^2 = u^2 + 2as$ enables v to be found. Almost all candidates made a good attempt at this part.

(ii) In this part of the question the resistance force is given and the only other force acting on the particle is its weight. The method is to use Newton's second law to determine the acceleration (deceleration) and then use the constant acceleration equations to find the time taken for the particle to come to rest. When applying Newton's law there are two forces, namely the weight $0.4g$ acting downwards and the 5.6 N resistance force acting upwards. These forces produce a deceleration and then this can be used to find the required time. A common error seen was that candidates forgot the downward force due to the weight. On the whole most candidates made a good attempt at this question.

(iii) The velocity-time graph using co-ordinates (t, v) consists of a line from (0, 0) to (0.6, 6) and a line from (0.6, 6) to (2.1, 0) with all relevant values shown on the axes. Most candidates attempted this part but often had curves instead of straight lines and did not correctly annotate their axes.

Question 7

(i) In order to determine the required value of OP, it is first necessary to find the value of t when the particle is at instantaneous rest by solving $v = 0$. This gives the initial value of $t = 0$ but also the time at which the particle reaches P which is $t = 5$. In order to find the distance required, it is necessary to integrate the given expression for velocity. When this has been done, using the value of $t = 5$ produces the required answer. As there is a given answer care must be taken to show all working. Most candidates knew that they should integrate the expression for v but some made errors in the integration. Other candidates incorrectly used the constant acceleration equations when this was not the case.

(ii) In this part the displacement s is given and so using $s = 6.25$ when $t = 5$ in this expression immediately gives an equation relating k and c. Since the velocity is given at $t = 5$ then it is necessary to differentiate the given expression for s in order to find v. Once this has been achieved, use of $v = 1.25$ when $t = 5$ produces a second relationship between k and c. These equations can then be solved simultaneously to find k and c. Although most candidates differentiated correctly some did not use the fact that $OP = 6.25$ and so could not solve their single equation.

(iii) In this part it was necessary to differentiate the expression for v to obtain the acceleration. Once this had been done a simple substitution of $t = 5$ into this expression gave the required result. Again some common errors involved the use of constant acceleration equations.

MATHEMATICS

Paper 9709/42
Paper 42

Key messages

Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Cases where this was commonly not adhered to were seen in **Question 3**, **Question 4**, **Question 6** and **Question 7**. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.

When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation or a work/energy equation. Such a diagram would have been particularly useful for **Question 3**, **Question 6** and **Question 7**.

In questions such as **Question 1**, where velocity is given as a cubic function of time, it is important to identify that calculus must be used and that it is not possible to apply the equations of constant acceleration.

General comments

The paper was generally very well answered by many candidates, although a wide range of marks was seen.

Most candidates work was well presented, but candidates must be reminded of writing their answers clearly in either a black or dark blue pen.

In **Question 6**, the angle θ was given exactly as $\cos\theta = \frac{24}{25}$. In cases such as this, candidates should be reminded that they do not need to evaluate the angle, as this type of problem can often lead to exact answers and so any approximation of the angle can lead to a loss of accuracy.

The examination allowed candidates at all levels to show their knowledge of the subject, whilst still differentiating well between the stronger candidates. **Questions 1** and **2** were found to be the most accessible whilst **Questions 4** and **5** proved to be the most challenging.

Comments on specific questions

Question 1

In this question the displacement is given as a cubic function of time. The constant acceleration equations cannot be used here. Since the velocity is required it is first necessary to differentiate *s* to find the velocity, *v*. It also necessary to differentiate *v* in order to find the acceleration, *a*. In order to answer the question, the expression for *a* must be set equal to zero and then solved for *t*. This value of *t* is now used in the expression for *v*. Most candidates made a very good attempt at this question but some who found the value to be negative gave their answer as a positive. Since the question asked for velocity and not speed, the negative is the correct answer.

Question 2

(i) Most candidates answered this question correctly. The acceleration is given between $t = 30$ and $t = 35$ and one method is to equate the given acceleration to the gradient of the line between those two points to find *V*. This was the technique used by the majority of candidates.

(ii) Since the total distance travelled is given, the most common method used was to evaluate the area under the velocity-time graph. This can be broken down in a number of different ways involving triangles, rectangles and trapezia. One of these regions can only be expressed in terms of U. When the sum of the areas is equated to 375, this leads to an equation in U. Although most candidates answered this question correctly, the most common errors seen involved the numerical calculations of the areas.

Question 3

This is a question where the other angles in the right angled triangle are not given directly. However it should be clear that is the 3, 4, 5 triangle such that sin PAB = 0.8 which means that there is no need to evaluate the angles in the triangle. Despite this most candidates found the angles as 36.9 and 53.1 but there is a danger that this will lead to a lack of accuracy. There are many different possible approaches to this problem. The most straightforward method is to resolve forces vertically and horizontally. This leads to two equations relating the tensions T_A and T_B in the two strings. The solutions of these simultaneous equations will produce the required answer. A common error was to disregard the weight of the particle. Other common errors seen were to either wrongly assume that the tensions in the two strings were the same, or to assume that the other two angles in the triangle were both 45°.

Question 4

(i) In this question it is necessary to use the given information in the relationship $P = Fv$ to determine the power required to maintain a constant given speed. Since the lorry is travelling at a constant speed, the resistance force acting on the lorry must exactly balance the driving force. hence the driving force is 3000 N. Most candidates found this correctly and used it to find the required power.

(ii) This question specifically asks for the problem to be solved using an energy method and full marks could not be gain unless this advice was followed. Using the given information, it is possible to determine the loss in kinetic energy and the gain in potential energy as the lorry moves from A to B. Since the question asks for the height of B above A, either this value or the distance travelled up the plane must be used in the equation. The energy equation will then take the form
KE loss = PE gain + WD against resistance. The only unknown in this equation will be the height of B above A which will lead to the required result. Many candidates did not multiply the resistance force of 3000 N by the distance moved up the plane in order to find the work done against resistance. Another error seen was to have incorrect signs in the energy equation.

Question 5

(i) This question was found by most candidates to be the most difficult on the paper. It requires some thought as to how to approach it and again there are several different methods of approach. Using the constant acceleration equations for the motion of A, the distance it has travelled t seconds after it was projected is $20t - \frac{1}{2}gt^2$. When considering particle B, t seconds after A is released particle B has travelled a distance $\frac{1}{2}g(t-1)^2$. Many candidates did not take into consideration the time lag between the times of projection. Once these two expressions have been found they will then collide when the sum of the two distances equals 40 m. However, many candidates thought that they would meet when these two distances were equal. An alternative approach is to find the distance travelled by A one second after projection and to find its velocity. The motion of the two particles can now be modelled using the same t value. Once the time at which collision occurs is found then the height at this time can be determined.

(ii) In this part of the question the time at which the collision occurred can be used in an equation of the form $v = u + at$ for each particle. It must be remembered that the times used will differ by one second. Candidates who did not find the correct solution in **part (i)** were still able to earn the method marks in this part. The most common error was to use the same t value for both particles.

Question 6

(i) In this question it is given that a constant force is applied to the block and so the constant acceleration equations may be used. From the given information we have, $u = 0$, $s = 4.5$ and $t = 5$ and using the equation $s = ut + \frac{1}{2}at^2$ the acceleration can be determined. Once the acceleration is known then Newton's second law of motion can be applied. The forces acting on the block are a component of the 6 N force and the opposing friction force. This equation will give the required value of the friction force. Most candidates successfully found the acceleration but often used the 6 N force rather than a component of it when applying Newton's second law.

(ii) In this part of the question the value of μ is given correct to 3 significant figures and so care must be taken to show all working. As the friction force is known, all that is now required is to find the normal reaction, so that the equation $F = \mu R$ can be used to find μ. When resolving vertically there are three forces involved, the weight, a component of the 6 N force and the normal reaction and hence the normal reaction, R, is a combination of the weight and a component of 6 N. Many candidates incorrectly stated that $R = 3g$ and were then unable to achieve the given result. Another common error was a misreading of the question and because an angle was given, several candidates assumed that the motion took place on an inclined plane rather than on the horizontal as stated.

(iii) Many candidates followed through the incorrect friction force from **6(i).** In this part the 6 N force had now been removed and so $R = 3g$ is correct and so the friction force is now $F = 0.165 \times 3g$. This is the only force now acting on the block and leads to an acceleration of $0.165g$ and this can be used in the constant acceleration formula $v = u + at$ and the time taken to come to rest can be found. Many candidates correctly obtained this value but in the question it asks for the total time in motion which requires the extra 5 seconds to be added.

Question 7

(i) This question involves a system of connected particles. There are three possible equations of motion and any two of them will enable the problem to be solved. Newton's second law can be applied either to particle P or to particle Q or to the system of both particles. The system equation will not involve the tension in the string. Solving any two of these three equations will give the tension and the magnitude of the acceleration. A common error made by candidates was to assume that the force acting on P was $0.3g$ rather than a component of the weight down the plane. Most candidates made a very good attempt at this part of the question.

(ii) The constant acceleration equation $s = ut + \frac{1}{2}at^2$ can be used here to find the required time since $u = 0$, $s = 0.8$ and $a = 0.4$ as found in **part (i)**. A common error was for candidate to use $a = g$ in this equation.

(iii) Once the string becomes slack particle P will continue to move up the plane until it comes to rest and will then return to the original position when the string will again become taut. There are various methods which could be used. One method is to determine the time taken before P comes to rest and then the total time will be found by doubling this. The speed of P as the string becomes slack is simply $v = at = 0.8$ The acceleration of P while it is moving up the plane will be a component of g namely $-g \sin \theta = -6$. This can be used in the equation $v = u + at$ to find t. An alternative is to use the equation $s = ut + \frac{1}{2}at^2$ with $s = 0$, $u = 0.8$ and $a = -6$ and this will give the total time that the string is slack. As the question asks for the time from the instant that P is released and so it must be remembered to add the 2 seconds found in **part (ii)**.

MATHEMATICS

Paper 9709/43
Paper 43

Key messages

Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.

When answering a 'show that' question where the answer is given, candidates are advised to ensure that sufficient working is shown, this was required for **Question 4(i)**, **Question 6(ii)** and **Question 7(ii)**.

General comments

Much of the candidates' responses were of a very high standard, with clearly written and accurate solutions. The most accessible questions were found to be **Question 1**, **Question 3** and **Question 5(i),** whilst **Question 4(i)**, **Question 6(ii)** and **Question 7 parts (ii)** and **(iii)** were the most challenging.

Comments on specific questions

Question 1

This was a relatively straightforward question which was answered correctly by almost all candidates. Of the few errors seen, these included the application of $F = \mu R$ with $R = 500$ instead of $500g$, and the application of $R = \mu F$ instead of $F = \mu R$.

Question 2

This question was found to be more challenging for candidates, since the resisting force was not given as constant, the most appropriate solutions made use of a work energy approach. The potential energy gain and change in kinetic energy were often found correctly. However, in forming an energy equation, some candidates omitted a term for example the work done by the driving force. Other responses included an extra term, for example, the work done by the weight component in addition to the potential energy gain. Some candidates interpreted the distance 500 m to be the increase in height rather than the distance moved along the plane. The number of zeros involved in the various quantities sometimes led to numerical errors.

Question 3

Most candidates identified what was required, resolving the forces horizontally and vertically and then attempting the application of trigonometry and Pythagoras' Theorem. Many candidates found R and α successfully. Those who worked with $R\sin\alpha$ and $R\cos\alpha$ sometimes showed both as negative, rather than positive leading to a negative value for R. Other errors included the omission of a component in one or both directions, or an error with sin or cos, for example, $100\cos\alpha$ instead of $100\sin\alpha$ horizontally. 50.7 (= 90 – α) was occasionally seen mistakenly for α. Slight inaccuracies such as $R = 89.8$ or $R = 90.0$ sometimes resulted from premature approximation in working.

Question 4

(i) This was a more challenging question with many possible approaches to finding the acceleration depending on the use of different '*suvat*' formulae. Those who applied $s = ut + \frac{1}{2}at^2$ to *PQ* and *QR* were often successful, although the velocity at *P* was sometimes mistakenly taken to be 0 ms^{-1} rather than 20 – 10*a*. Those who applied $s = vt - \frac{1}{2}at^2$ for *PQ* were able to avoid the unknown velocity at *P*. Some candidates misinterpreted the relation between *PQ* and *QR* using *PQ* = 1.5*QR* instead of *QR* = 1.5*PQ*. The given answer of 0.8 ms^{-2} sometimes appeared following incorrect work. Occasionally *a* = 0.8 was used inappropriately as part of the solution to show *a* = 0.8, for example in calculating and then using the velocity at *P* as 20 – 0.8 × 10.

(ii) Following **part (i)**, the solution for **part (ii)** was found to be more straightforward for work out. The shortest solutions considered the 20s motion from *Q* to *S* rather than splitting this into *QR* and *RS*. Some candidates mistook which distance was required and found e.g. *PS* or *PR* with a consequent error in calculating the average velocity. Occasionally candidates who had been unsuccessful in **part (i)** omitted **part (ii)**, perhaps not realising that the given answer of *a* = 0.8 could enable them to answer **part (ii)** accurately.

Question 5

This question was usually well answered with the first two parts very often fully correct.

(i) This part was often well answered. Occasional sign errors were seen, such as $\frac{240}{6} = 80 \times 0.3 - R$, leading to *R* = –16 N.

(ii) The majority of candidates equated the driving force $\frac{240}{v}$ to the resistance 16 N as required. The erroneous value of 10 ms^{-1} was sometimes obtained from equating the driving force to either $\frac{240}{6} - 16$ from **part (i)** or '*ma*' from **part (i)** without recognising that *a* = 0 for steady speed.

(iii) Candidates often attempted to apply *F* = *ma*, they were mostly successful in doing so. The usual errors seen were a missing weight component or driving force; a sign error in the equation; or 80 sin3 used instead of 80 g sin3. A common error was to give a one or two significant figure answer (0.03 or 0.027) instead of rounding to the required three significant figures.

Question 6

(i) Most candidates knew that simultaneous equations in *c* and *k* were required and usually formed one equation accurately using *v* = 10 when *t* = 5. Many candidates appropriately used $s = \int v\, dt$ to form a second equation in *c* and *k*, but some applied both $a = \frac{dv}{dt}$ and *v* = *u* + *at* to obtain an equation in *c* or *k*, despite the varying acceleration. A few candidates included a constant of integration without applying *t* = 0, *s* = 0, thus forming an equation with three unknowns. Candidates were usually able to obtain a solution to their simultaneous equations suitably and often accurately.

(ii) **Part (ii)** was less well-answered. Many candidates found $\frac{dv}{dt}$ and then used the given *t* = 2.5 to obtain the acceleration at this time. Other candidates found $\frac{da}{dt}$ and equated to zero as expected, but if *c* was incorrect from **part (i)** their equation did not lead to *t* = 2.5. Those who found an incorrect $\frac{da}{dt}$ and substituted *t* = 2.5 sometimes concluded erroneously, for example that $\frac{da}{dt} > 0$ implies a minimum. Several candidates completed the square as an alternative method of solution which was often successful provided that their value of *c* was correct.

Question 7

(i) Most candidates were able to write down equations of motion for particles *A* and *B*, occasionally equating to *ma* instead of *kma* for particle *B*. Those who used a '*suvat'* formula to find $a = 2$ and then used this to find *k* had an easier solution than those who found *a* in terms of *k* first. A few candidates were unable to progress beyond the equations of motion, not identifying that the acceleration could be found using the information given about the distance and time taken to reach the ground. Since the equation for particle *A* was independent of *k*, this equation was suitable for showing that $T = 12m$ even if the value of *k* found was incorrect.

(ii) Whilst it was straightforward to obtain the 'break' velocity, some candidates assumed $u = 0$ for particle *A* at the break and were thus unable to obtain the given 5.97 ms^{-1}. Others mistook the distance to the ground and solved $v^2 = 1.8^2 + 2g \times 0.81$. The time taken from 'break' to ground proved more difficult with some calculating 0.417 s, the time taken to fall 0.81×2 m, or 0.597, the time taken from the maximum height, whilst others included 0.9s of connected motion.

(iii) **Part (iii)** was found to be challenging for candidates. A considerable variety of straight line and curved graphs were seen, as well as several non-attempts. Candidates were expected to recognise two stages of motion, each represented by a straight line, due to the constant accelerations. It was also expected that the two key velocities and two key times would be shown. Whilst many represented the connected motion by a suitable straight line, the motion after the break was sometimes represented by a straight line ending at the t-axis and sometimes by two straight line segments with, for example, the velocity decreasing to zero and then increasing again.

MATHEMATICS

Paper 9709/51
Paper 51

Key points

Candidates should be reminded to always give answers to 3 significant figures unless otherwise stated in the question. As stated in the rubric on the front cover, $g = 10$ should be used not 9.8 or 9.81.

If $\sin\theta = \frac{3}{5}$ is given in the question then candidates should recognise that $\cos\theta = \frac{4}{5}$ and $\tan\theta = \frac{3}{4}$ and use these rather than find the value for θ, as this can result in a loss of accuracy.

General comments

Most candidates work was neat and well presented. Candidates should always refer to the formula booklet provided if in doubt of a specific formula.

Candidates should be reminded that an answer should be given to 3 significant figures unless otherwise stated in the question. This means that they should work to at least 4 significant figures.

The questions that candidates found more accessible were **1**, **2(i)**, **3(i)**, **4(i)** and **4(ii)**, whilst the questions found more challenging were **5(ii)**, **6(ii)**, **7(i)**, **7(ii)** and **7(iii)**.

Comments on specific questions

Question 1

This question was generally well answered. Some candidates did not appear to know that the centre of mass of a solid cone is a quarter of the height from the base.

Question 2

(i) This part of the question was usually well answered. To answer this question successfully candidates needed to use $v = u + at$ vertically.

(ii) This part of the question was generally well answered.

Question 3

(i) This part of the question was well answered.

(ii) Most candidates realised that it was necessary to integrate the equation from **part (i)**. Having integrated, the constant of integration, c, was found by putting $x = 0.8$ and $v = 3$. Some candidates used $x = 0$ and $v = 3$. Once c had been found, the resulting expression had to be manipulated.

Question 4

(i) This part of the question was generally well answered. Candidates needed to resolve horizontally and vertically. This gave 2 equations in terms of t and by eliminating t, the required trajectory equation was found.

(ii) This part of the question was also generally well answered. Candidates needed to put $y = x$ into the trajectory equation found in **part (i)**.

Question 5

(i) This part of the question was generally well answered. Candidates were firstly required to find the tension in the string at *A*, by using $T = \frac{\lambda x}{L}$. Having found *T*, Newton's Second Law could be used to find the required acceleration.

(ii) This part of the question proved to be challenging for several candidates. The first thing for candidates was to identify that the maximum speed would occur at the equilibrium position. This point can be found by using $T = \frac{\lambda x}{L}$, a 4-term energy equation then needed to be established.

Question 6

Candidates should be advised to draw a clear diagram to help with answering this question, and questions similar to this.

(i) This part of the question was generally well answered. The radius of the circle could be calculated by using Pythagoras's Theorem and the speed then found by using $v = r\omega$.

(ii) This part of the question proved to be quite challenging for many candidates. Two equations in T_{PA} and T_{PB} needed to be found. This was done by resolving vertically and by using Newton's Second Law horizontally for *P*. From these equations T_{BP} could be found. The final step to find the natural length required the use of $T = \frac{\lambda x}{L}$.

Question 7

Candidates should be advised to draw of a clear diagram to help with answering this question, and questions similar to this.

(i) This part of the question could be solved by firstly using the similar triangles *MGB* and *MCE* with $GB = 0.3$ m. This led to $ME = 0.6$ m and then finally by using triangle *ACE*, the required angle could be found.

(ii) By using triangle *ACE*, *AC* could be calculated. The required answer followed by taking moments about *A*.

(iii) Candidates needed to resolve horizontally and vertically to find the friction force, F and the normal reaction, R. When found, $F = \mu R$ was applied to give μ, the coefficient of friction.

MATHEMATICS

Paper 9709/52
Paper 52

Key points

Candidates should be reminded to give answers to 3 significant figures unless otherwise stated in the question. As stated in the rubric on the front cover, g = 10 should be used not 9.8 or 9.81.

If $\sin\theta = \frac{3}{5}$ is given in the question then candidates should recognise that $\cos\theta = \frac{4}{5}$ and $\tan\theta = \frac{3}{4}$ and use these rather than find the value for θ, as this can result in a loss of accuracy.

General comments

Most candidates work was neat and well presented. Candidates should always refer to the formula booklet provided if in doubt of a specific formula.

Candidates should be reminded that an answer should be given to 3 significant figures unless otherwise stated in the question. This means that they should work to at least 4 significant figures.

The questions that candidates found more accessible were **1**, **2(i)**, **2(ii)**, **6(i)** and **6(ii)**, whilst the questions found more challenging were **5(ii)**, **6(iii)** and **7(iii)**.

Comments on specific questions

Question 1

This question was quite well answered by many candidates. Some candidates tried to use $T = \frac{\lambda x}{L}$, which was an incorrect method. In order to answer the question correctly, the most successful approach was to use an energy method.

Question 2

Both parts of this question were generally well answered.

Question 3

Many candidates were successful in answering this question. The most efficient method for answering this question was to use Newton's Second Law vertically and then apply a 4-term energy equation.

Question 4

(i) This part of the question was generally well answered. Candidates needed to use $\tan 30° = \frac{v_y}{v_x}$, rather than $\tan 30° = \frac{y}{x}$, and also $v_y = 30 - V\sin 60°$ should have been used rather than $v_y = V\sin 60° - 30$.

(ii) This part was generally well answered. Candidates were required to find x and y after 3 seconds. Pythagoras' theorem then resulted in the required distance.

Question 5

(i) To answer this part the first step was for candidates to find the radius of the circle which was 0.5 sin 30 = 0.25 m. The next step was to resolve vertically in order to find the tension. Once the tension was found, Newton's Second Law horizontally gave the required velocity. Many candidates provided good answers to this part of the question.

(ii) To solve this part of the question candidates were required to resolve vertically and also to use Newton's Second Law horizontally. By solving the two equations, the two tensions could be found. Many candidates appeared to find this part of the question more challenging.

Question 6

(i) Most candidates answered this part of the question correctly.

(ii) A small number of candidates did not solve the equation $0.45x^{\frac{1}{2}} - 1.5 = 0$. Overall, this part of the question was well answered.

(iii) Many candidates found this part of the question challenging. The first step was to integrate the equation from **part (i)**. This often resulted in v =... and not $\frac{v^2}{2} = \ldots$. For candidates who acquired the correct integration it was then necessary to substitute x = 11.1, the value found in **part (ii)**, this gave the constant of integration, c, to be $\frac{50}{9}$. Then by substituting $x = 0, \frac{v^2}{2} > \frac{50}{9}$ to result in $v > \frac{10}{3}$.

Question 7

(i) This part of the question was generally well answered. To be successful in answering this question candidates were required to take moments about the line *AD*.

(ii) Many candidates correctly answered this part of the question.

(iii) This part appeared to be a challenging question for candidates. Candidates needed to recognise that *AD* would make an angle of 40° or 20° with the vertical. Candidates then needed to take moments about *A* for both situations. This resulted in $W \times AG \sin 10° = 7 \times 2.4 \cos 40°$ and $W \times AG \sin 10° = 7 \times 2.4 \cos 20°$.

MATHEMATICS

Paper 9709/53
Paper 53

Key points

Candidates should be reminded to always give answers to 3 significant figures unless otherwise stated in the question. As stated in the rubric on the front cover, $g = 10$ should be used not 9.8 or 9.81.

If $\sin\theta = \frac{3}{5}$ is given in the question then candidates should recognise that $\cos\theta = \frac{4}{5}$ and $\tan\theta = \frac{3}{4}$ and use these rather than find the value for θ, as this can result in a loss of accuracy.

General comments

Most candidates work was neat and well presented. Candidates should always refer to the formula booklet provided if in doubt of a specific formula.

Candidates should be reminded that an answer should be given to 3 significant figures unless otherwise stated in the question. This means that they should work to at least 4 significant figures.

The questions that candidates found more accessible were **1**, **2(i)**, **3(i)**, **4(i)** and **4(ii)**, whilst the questions found more challenging were **5(ii)**, **6(ii)**, **7(i)**, **7(ii)** and **7(iii)**.

Comments on specific questions

Question 1

This question was generally well answered. Some candidates did not appear to know that the centre of mass of a solid cone is a quarter of the height from the base.

Question 2

(i) This part of the question was usually well answered. To answer this question successfully candidates needed to use $v = u + at$ vertically.

(ii) This part of the question was generally well answered.

Question 3

(i) This part of the question was well answered.

(ii) Most candidates realised that it was necessary to integrate the equation from **part (i)**. Having integrated, the constant of integration, c, was found by putting $x = 0.8$ and $v = 3$. Some candidates used $x = 0$ and $v = 3$. Once c had been found, the resulting expression had to be manipulated.

Question 4

(i) This part of the question was generally well answered. Candidates needed to resolve horizontally and vertically. This gave 2 equations in terms of t and by eliminating t, the required trajectory equation was found.

(ii) This part of the question was also generally well answered. Candidates needed to put $y = x$ into the trajectory equation found in **part (i)**.

Question 5

(i) This part of the question was generally well answered. Candidates were firstly required to find the tension in the string at A, by using $T = \frac{\lambda x}{L}$. Having found T, Newton's Second Law could be used to find the required acceleration.

(ii) This part of the question proved to be challenging for several candidates. The first thing for candidates was to identify that the maximum speed would occur at the equilibrium position. This point can be found by using $T = \frac{\lambda x}{L}$, a 4-term energy equation then needed to be established.

Question 6

Candidates should be advised to draw a clear diagram to help with answering this question, and questions similar to this.

(i) This part of the question was generally well answered. The radius of the circle could be calculated by using Pythagoras's Theorem and the speed then found by using $v = r\omega$.

(ii) This part of the question proved to be quite challenging for many candidates. Two equations in T_{PA} and T_{PB} needed to be found. This was done by resolving vertically and by using Newton's Second Law horizontally for P. From these equations T_{BP} could be found. The final step to find the natural length required the use of $T = \frac{\lambda x}{L}$.

Question 7

Candidates should be advised to draw of a clear diagram to help with answering this question, and questions similar to this.

(i) This part of the question could be solved by firstly using the similar triangles MGB and MCE with $GB = 0.3$ m. This led to $ME = 0.6$ m and then finally by using triangle ACE, the required angle could be found.

(ii) By using triangle ACE, AC could be calculated. The required answer followed by taking moments about A.

(iii) Candidates needed to resolve horizontally and vertically to find the friction force, F and the normal reaction, R. When found, F = μR was applied to give μ, the coefficient of friction.

MATHEMATICS

Paper 9709/61
Paper 61

Key messages

In order to be successful in this component, candidates should understand all the topic areas that are stated in the syllabus.

The use of simple diagrams can assist understanding in many questions by helping to visualise the conditions stated within the question and should be encouraged.

Communication of processes by showing workings assists both the candidate to check their work and allows for method marks to be awarded credit when there are numerical errors in calculations.

General comments

Candidates appeared to have sufficient time to complete the paper. Some candidates did not appear to be well prepared for the component as several questions were not attempted.

Questions 1, **3** and **4** were generally answered well while **Questions 6** and **7** appear to have been more challenging for candidates.

It was noted that several candidates appeared to have made errors within their calculations because of misreading their own writing. Candidates should ensure that their presentation is clear such that they can check their work and that marks can be awarded appropriately.

Comments on specific questions

Question 1

Although there was no requirement to construct a tree diagram, it is a significant aid to understanding the context of the question and often better solutions included one. Where attempted, most candidates made some progress towards calculating *x*, although algebraic and arithmetical slips were noted within solutions.

Question 2

(i) This was a typical Binomial distribution question. Good solutions recognised that the most efficient approach was to subtract the terms that were not required from 1, and clearly stated these prior to evaluation. A less efficient approach was to sum the 8 terms that did fulfil the condition. Candidates should be encouraged to understand the vocabulary used within these contexts, as a common error was to include 8 within the required group.

(ii) This question was found challenging by most candidates. The question builds upon Binomial distribution knowledge and requires candidates to recognise that ‘at least one’ is the compliment of ‘none’. Trialling values was a common approach to solving the exponential equation, although the use of logs was also evidenced.

Question 3

(i) Candidates with a good understanding of the basic formula for mean $\left(\frac{\sum x}{n}\right)$ and variance $\left(\frac{\sum x^2}{n} - \mu^2\right)$ identified that this question involved substituting the given values into the formulae and solved for the remaining unknown. Some poor algebraic manipulation was evidenced in solutions; candidates should be reminded that knowledge of Pure Mathematics 1 will sometimes be required.

(ii) Few correct solutions to this question were seen. Where attempted, solutions were often summing the mean and variances for the two data groups. Better solutions recognised that a total for x and x^2 for the 30 values needed to be found before being substituted into the appropriate formulae.

Question 4

(i) Good solutions recognised that the probabilities in the probability distribution table sum to 1 and used this property to form an equation and solve for p. Candidates should be encouraged to be accurate when solving equations as arithmetical slips with fractions were noted. Some candidates used decimals throughout, which is acceptable.

(ii) Where candidates found a value for p, they were usually confident in using the appropriate formula for E(X) and Var(X). Good solutions included unsimplified expressions to assist with communicating the process and enabling candidates to check their work later for accuracy. A common error was not squaring the mean within the variance formula.

(iii) Most candidates found this question challenging. A conditional probability was required, with the values required for the numerator and denominator being obtained from the probability distribution table. A common error was to state the probability that X was equal to 2 directly from **4(i)**.

Question 5

For this component, candidates are expected to use probability graphs accurately, interpreting any scale given precisely to find exact values, wherever possible. The use of a ruler to assist with this is often helpful, although it was noted that many graphs had freehand lines to assist with identifying values that were being used.

(i) Good solutions clearly marked 14 and 24 on the horizontal axis, drew vertical lines to the cumulative frequency curve and using horizontal lines found the appropriate cumulative frequencies on the vertical axis. A simple calculation was then shown to find the difference between the two values. Many candidates used alternative approaches, sometimes counting the grid squares and stating this as the answer.

(ii) The best solutions calculated 10 per cent of the 160 palm leaves and identified the cumulative frequency for 90 per cent of the leaves being found. Weaker solutions either stated 16, or stated the length which had less than 10 per cent of the leaves. A significant number of candidates appeared to have disregarded the scale on the vertical axis and did not read off at their stated value. Good solutions showed clear construction lines on the graph.

(iii) While many candidates attempted the median successfully, calculation of the interquartile range was often omitted, with weaker candidates stating only the upper and lower quartiles. Candidates must ensure that they take care using the scale as this can affect the accuracy of answers.

(iv) The question required candidates to compare the data within context; candidates were directed to identify the differences between the central tendencies and spread for the two sets of data. Good solutions identified that the median was the appropriate central tendency and included a comparative statement, such as ‘the median for Ransha was higher’. Good solutions also recognised that the range was not a suitable comparison for spread, as they were equal, but that the interquartile range calculated in **5(iii)** could be compared with the interquartile range identified within the box-and-whisker plot. Weaker solutions only stated the values, and as this does not constitute a comparison no credit could be awarded.

Question 6

The use of simple diagrams throughout this question assisted candidates to interpret the conditions required.

(i) Successful solutions often contained a simple diagram with the four Es together, which helped identify that there were only 9 individual items to arrange and there were only the repeated effect of the two Ss needed to be removed. Some solutions included an extra term that calculated the number of ways the four Es could be arranged including the effect of the repeated letters. As the number of arrangements is an exact value, it should not be rounded to 3 significant figures.

(ii) Some good solutions using both the subtraction or insertion approaches were seen. The better solutions using the insertion approach included a simple diagram to clarify the conditions, recognised that there was the effect of the four Es to eliminate as they were not identifiable and that when the Ss were inserted they could be interchanged and so needed to treated as a combination rather than a permutation. The better solutions using the subtraction approach clearly stated the total number of arrangements and arrangements with the Ss together were being calculated, and these evaluated before the final subtraction. A common error was to not eliminate the effect of the repeated letters in either expression.

(iii) Many candidates attempted this question, although few fully correct solutions were seen. Good solutions identified the possible scenarios that fulfilled the requirements before calculating the number of selections that were possible for each scenario. A common misconception was that if SE was chosen, then the remaining S and Es would be included with the remaining letters for selection in the remaining places. Many candidates treated the Ss and Es as if they were individually identifiable and multiplied appropriately to include this within their calculation. The omission of SEEE was not uncommon with a final total of 41 being stated.

Question 7

The use of a simple sketch of the normal distribution curve can be helpful to identify the required probability area and the best solutions often included one.

(i) Although this appeared to be a more challenging normal distribution context, good solutions recognised that the normal standardisation formula needed to be applied for boundary conditions and the appropriate probability area found. Better solutions did not use a continuity correction as time is a continuous variable. Many candidates did not identify the required area, and probabilities greater than 1 were again identified. Some simple arithmetical errors were noted in the evaluation of the probabilities from the z-value using the tables.

(ii) Many candidates found this question challenging. Solutions with a simple sketch were often more successful at identifying the required probability area, and hence determining that the *z*-value was negative. A common error was to treat 0.92 as a *z*-value and then find the probability from the tables to form the equation.

(iii) Many candidates did not attempt this question. Better solutions recognised that 46 seconds was one of the boundaries used in **7(i)** and found the compliment to provide the probability value required. Weaker solutions simply recalculated the required value. Good solutions then used the Binomial distribution to calculate the probability of fulfilling the condition.

MATHEMATICS

Paper 9709/62
Paper 62

Key messages

Candidates should be encouraged to use simple diagrams as to aid understanding in many questions by helping to visualise the conditions stated within the question.

It is essential to work to at least 4 significant figures throughout a question to ensure that the required accuracy can be achieved. Many solutions did not achieve the required accuracy of the questions as many candidates had worked to 3 significant figures or 3 decimal places earlier in the process.

Communication of processes by showing workings assists both the candidate to check their work and allows for method marks to be awarded credit when there are numerical errors in calculations.

General comments

Candidates appeared to have had sufficient time to complete the paper, although some candidates would appear to have spent a significant amount of time on **Question 3**. The use of a ruler is expected when drawing statistical diagrams such as a histogram.

Questions 4, 5 and 6 were generally answered well, while **Question 2 and 7** appear to have been more challenging for candidates.

It was noted that several candidates appeared to have made errors within their calculations because of misreading their own writing. Candidates should ensure that their presentation is clear such that they can check their work and that marks can be awarded appropriately.

Comments on specific questions

Question 1

(i) Many candidates were successful in attempting this question. Good solutions included reordering the data to facilitate identification of the median and quartiles. Most solutions correctly stated the value for the median, although some adjusted their value to coincide with an original data value. A significant number of solutions did not identify the quartiles accurately by not finding the mid-values of the data above or below the median. A number of solutions were also adjusted to coincide with original data values. However, the interquartile range was attempted by almost all candidates. Weaker solutions did not reorder the data initially and found the mid-value and quarter-values of the question data.

(ii) Good solutions clearly stated that 110 was significantly different from the other data values and would make the mean unsuitable. Weaker solutions stated that extreme values affect the mean, however did not link this to the question context, therefore candidates could not gain credit.

Question 2

(i) Although there was no requirement to construct a tree diagram, it is a significant aid to understanding the context of the question and better solutions often started with one. Almost all candidates made some progress towards calculating the value of *x* but did not always included sufficient method for a 'show' question, where a simple two term equation was necessary to justify the given answer.

(ii) Better solutions recognised that the conditional probability calculation could be based upon the information provided in **part (i)**. A small number of good candidates recalculated the probabilities using the standard textbook approach. Weaker responses used the complements of the required probabilities or did not calculate the probabilities for all the necessary outcomes required for the denominator and so gained little credit.

Question 3

(i) Many candidates found this question challenging. Good solutions made effective use of the space around the data table to calculate both the class width and the frequency density. Since speed is continuous, the upper/lower boundaries of the classes needed to be used so the best solutions used the main grid-lines on the horizontal axis for values 9·5, 19·5, 29·5 etc. rather than 10, 20, 30 etc. This was acceptable and enabled more accurate drawing of the column lines. Good solutions also showed the axes fully labelled with 'frequency density' and 'speed, km/h'. A common misconception was that the class width was the difference between the table values. A small number of candidates calculated the frequency density as 'class width/frequency'. A large proportion of candidates' graphs included correctly labelled axes, rather than simply stating either 'speed' or 'km/h'. It would be advised for candidates to choose scales that enable graphs to be drawn accurately. A number of speed scales, e.g. 3 cm: 20 km/h or 1 cm: 20 km/h, made it difficult for candidates to achieve the expected accuracy. Weaker solutions showed a bar chart of the data.

(ii) Many good solutions were seen to this question part. Stronger responses constructed a table to manage their calculation, stating the frequency, mid-value and their product before the final calculation. The most common approach was to produce a single calculation; inaccuracy in calculations were often noted here. A common error was to use either the class width or upper boundary in place of the midpoint. Several candidates summed their frequencies inaccurately however did not compare with the information within the question.

Question 4

(i) Almost all candidates used the Binomial distribution accurately. The most frequent error was to omit '8 households satisfied' when interpreting 'at least 8 are satisfied'. Candidates should be encouraged to develop an accurate understanding of the terminology used in this style of question. Many solutions did not gain full credit due to premature approximation. Candidates should be reminded that at least 4 significant figures are expected to be used in all work leading to the final answer.

(ii) Many good solutions were seen using the Normal distribution as an approximation to the Binomial distribution. Better solutions stated the calculations required for the mean and variance before evaluating, identified that a continuity correction was required because the data is discrete, and interpreted that 'more than 84' did not include 84 within the data. A simple sketch of the normal curve was often included to clarify the probability area required. A common error was to include 84, and either use of the lower bound or no continuity correction with the normal formula. A number of solutions with correct method did not gain full credit due to premature approximation, either in calculating the variance/standard deviation or in the evaluation of the formula itself.

Question 5

(i) Stronger responses usually contained a sample space diagram of the possible outcomes before any probability calculations or the construction of the probability distribution table. The use of a sample space diagram aids the interpretation of the question information and enables the probabilities to be stated without any calculation. Several candidates interpreted the question as requiring only positive scores were permitted, and so either found the difference between the spinners or ignored the '-1' and continued with a probability total of less than 1. Candidates should be encouraged to read questions carefully, as solutions involving a red spinner numbered 1, 2, 3, 4 or the values on the spinners summed were often seen.

(ii) Most candidates who produced a probability distribution table in **5(i)** used the appropriate method here to calculate the variance. Better solutions clearly stated the unsimplified calculation for E(X) and then Var(X), with the final value stated as an accurate fraction. Where the probability distribution table was incorrect in **5(i)**, credit could be awarded for the method provided the unsimplified expressions were stated to justify the approach. A common error was not to use $E(X)^2$ in the variance calculation. Weaker solutions calculated E(X) and either stated that as the variance or squared this value. Premature approximation resulted in some answers being outside the required tolerance.

Question 6

(i) Many good solutions were seen for this Normal distribution question. Better solutions often included a simple sketch to help identify the required probability area. A common error was arithmetical inaccuracy when adding the value for the third decimal place of the z-value to state the probability. Most candidates correctly identified that the data was continuous and so did not use a continuity correction.

(ii) Many candidates did not attempt this question. Candidates who drew a sketch of the Normal distribution often identified that this question used the symmetry properties of the distribution with the answer from **6(i)** to calculate the required probability. However, most candidates calculated at least one of the probabilities within their solution. Final probabilities greater than 1 were common and these should have indicated to candidates that they have made an error.

(iii) The question was found challenging by many candidates. The most successful solutions were logically presented, with equations clearly identified and formed and algebraic processes accurately applied. The use of appropriate pure mathematic techniques is expected within this paper, and these should include sufficient working to justify the answer. Weaker candidates either found the z-values inaccurately, often using the tables to find a probability or using the original probability when forming the initial equations. The use of a sketch of a Normal distribution curve was seen in the stronger responses as this assisted in identifying if the z-value was negative.

Question 7

The use of simple diagrams throughout this question assisted candidates to interpret the conditions required.

(i) Many solutions did not apply the additional conditions identified in the instructions but calculated the total number of different ways that the 9 letters could be arranged. The use of a simple diagram assisted the best solutions to identify that there were only 6 different items to arrange, although some solutions did include additional terms to calculate the number of ways the block of Ts or Os could be arranged.

(ii) Many good solutions using both the subtraction or insertion approaches were seen. Better solutions using the insertion approach included a simple diagram to clarify the conditions, recognised that there was the effect of the 3 Os to eliminate as they were not identifiable and that when the Ts were inserted they could be interchanged and so needed to treated as a combination rather than a permutation. Better solutions using the subtraction approach clearly stated the total number of arrangements and arrangements with the Ts together being calculated, and these evaluated before the final subtraction. A common error was to not eliminate the effect of the repeated letters in either expression.

(iii) Candidates should be reminded to ensure they read the question carefully, as it was evident in this question that many candidates did not as few probabilities were attempted. Stronger solutions had a simple diagram to clarify the condition, and used similar approaches as the previous parts to calculate the values required within the probability. Most solutions simply calculated the number of arrangements with the Ts placed at the either end of the letters. Alternative approaches are possible, and some candidates successfully used conditional probability here.

(iv) Most candidates attempted this question, although few fully correct solutions were seen. Good solutions identified the possible scenarios that fulfilled the requirements before calculating the number of selections that were possible for each scenario. A common misconception was that if

OOT was chosen, then the remaining O and T would be included with the remaining letters for selection in the remaining places. Many candidates treated the Os and Ts as if they were individually identifiable and multiplied appropriately to include this within their calculation. The omission of OOOTT was not uncommon with a final total of 14 being stated.

MATHEMATICS

Paper 9709/63
Paper 63

Key messages

The accurate use of tables is an expectation for this component and in questions such as **4(i)** it was expected that candidates would use the tables provided.

Candidates need to be aware that they must show all their working. Many candidates presented well-structured solutions to the questions involving several stages, particularly **Question 2(ii)**, **Question 3(i)**, **Question 5(iv)** and **Question 6(v)**.

General comments

The requirement that all answers should be given to 3 significant figures unless stated otherwise seems to have been followed by the majority of candidates. However, there were many instances of premature approximation, particularly in **Question 4(ii)** and **Question 7(i)(a)**. Candidates need to be aware that they should work with numbers to 4 or more significant figures before they arrive at their final answer, in order to achieve the accuracy required.

Candidates also need to be aware to use a 'sensible' scale for graph questions, which allows for plotting clear points. Those who used the scale 3 cm: 20 minutes in **Question 5(i)** rarely plotted all the points correctly and graphs were often unclear and lacked any labelling.

Comments on specific questions

Question 1

(i) Most candidates understood the question and answered it correctly. A few candidates assumed independence and arrived at the correct answer from multiplying P(male) by P(not piano). Other candidates found the probability that the randomly chosen candidate did not play the piano given that he was male; these candidates worked with a denominator of 160 instead of 300.

(ii) Stronger responses made their argument clear to their reader in answering this question. These candidates clearly labelled the probabilities that they were working with, substituted the values in a calculation, stated the required condition that had been met and clearly stated their conclusion.

The first method in the mark scheme was the preferred approach by most candidates but some candidates quoted the argument in terms of events A and B without any reference to the context of the question i.e. that the randomly chosen candidate is 'male' or 'does not play the piano'.

Those who assumed independence in **part (i)** generally found **part (ii)** more challenging. However, several candidates who misunderstood **part (i)** went on to produce a correct solution to **part (ii),** including the correct value for the probability of a candidate being male and not playing the piano.

Question 2

(i) This question was very well answered with almost all candidates knowing to divide 9! by the product of 2! and 3! to take account of the repeated letters. Few candidates forgot about the repeats while a small number found alternative more complicated but correct ways of arriving at the correct final answer.

(ii) Most candidates identified that they needed to consider separately the cases that began with D and ended with R or began with D and ended with O and then add the results. Many of these candidates arrived at the correct final answer and realised that the totals of $\frac{7!}{2!2!}$ and $\frac{7!}{3!}$ did not need to be multiplied by 2 or 3 as was seen in some scripts.

There were a number of successful attempts at other methods, the one most frequently seen being $\frac{7!}{3!2!}\times 5$ where they consider the middle seven letters and multiply by the 5 ways of having D at the front and R or O at the end.

Question 3

(i) The majority of candidates answered this question well, clearly identifying the three different scenarios that satisfied the conditions, knowing that they needed to multiply the three ${}^nC_r$ terms in each scenario and giving a clear presentation of how they arrived at their final answer. The most common errors were to omit one of the scenarios, usually 4 – 2 – 1, or to add the ${}^nC_r$ terms instead of multiplying before going on to add their number of ways for the three scenarios, arriving at a final answer of 99.

A number of candidates gained the Special Case mark where they incorrectly thought they could consider the three conditions (3 attackers, 2 defenders and 1 midfielder) in one go and multiply by 9, the difference between the total number of players and the 6 already chosen.

(ii) Stronger responses identified that once one car had been filled with either three or four team members, there was no choice about who went into the second car and that ${}^7C_3 = 35$ or ${}^7C_4 = 35$ gives the fully correct final answer. Several candidates then went on to unnecessarily multiply by ${}^3C_3$. Many other candidates did not appear to understand how to deal with two cars and either added or multiplied ${}^7C_3$ and ${}^7C_4$ and the incorrect final answer of 70 appeared frequently.

Question 4

(i) This question was answered well by the majority of candidates. Most were able to use the tables the correct way round and to equate the resulting z-value with a standardised expression using a mean of 148 and a standard deviation of 8.

(ii) This question was more challenging for candidates. Many of the stronger responses identified that the appropriate z-values were ± 0.5 without standardising. Others candidates correctly identified 148 and 152 as being the heights half a standard deviation above and below the mean and obtained the z-values ± 0.5 by standardising. Most of these two groups of candidates went on, successfully, to obtain the correct probability of falling within this range, 0.383.

Those who made sure to answer the question fully remembered to multiply their probability by 120 and to truncate their answer to a whole number of candidates. Some candidates mistakenly thought that 46.0 was a truncated answer.

Some candidates halved the standard deviation and worked with 4 instead of 8 while others worked with z = ±1 or ± 1.5.

Question 5

(i) Candidates who chose an appropriate scale made it much easier for them to plot a graph and generally did well in answering at this question. The two most straightforward and commonly seen scales on the 'Cumulative Frequency' axis were 1 cm:20 or 25 people and the most common scale on the 'Time in Minutes' axis was 1 cm:10 minutes. Some candidates clearly labelled their correctly scaled axes with 'Cumulative Frequency' and 'Time in Minutes', successfully did not to insert a scale break and plotted their points above the upper bounds, the first point being at (0, 0).

(ii) This question was well answered with most candidates reading a value from 100. Many candidates successfully gave an answer for the median within the required range, even if they had made small errors in drawing the graph. Some answers were given to a degree of exactness which indicated that other methods were being employed despite the request in the question to 'use the graph'.

(iii) This question was not very well answered, with most candidates confusing ‘at least T minutes’ with ‘fewer than T minutes’. Of the candidates who subtracted 80 from 200 and worked with 120 rather than 80, most obtained an answer within the required range or correctly followed through from their graph.

(iv) Candidates who obtained frequencies by subtracting the cumulative frequencies from each other and identified the mid-points generally went on to calculate the estimated mean correctly. Most candidates calculated six products and divided the sum by 200. There were however many candidates who used cumulative frequencies instead of frequencies, or used class widths, upper bounds or sometimes half the upper bounds instead of mid-points. Many candidates used mid-points 0.5 above the correct mid-points.

Question 6

(i) This question was very well answered with most candidates being awarded the mark.

(ii) Stronger responses appreciated that in a ‘show that’ question all working was required to be shown i.e. that there were two possible ways of the chosen balls being of different colours: red then white; white then red. They then gave the answer as the sum of two different products:
$\frac{3}{8}\times\frac{5}{7}+\frac{5}{8}\times\frac{3}{7}=\frac{15}{56}+\frac{15}{36}=\frac{15}{28}$.

Most candidates gave the minimum response i.e. the sum of the two products with no explanation. Doubling one of the products was not accepted as an adequate response in a ‘show that’ question where the answer is given.

(iii) A reasonable number of candidates identified the need for the Conditional Probability formula here and many gave a fully correct answer. Candidates were required to show how they arrived at the denominator value of $\frac{3}{8}$ for P(2^{nd} ball being red) by adding two products. Stronger responses identified that the numerator was their answer to **part (i)**, and their denominator was their answer to **part (i)** + P(white then red).

(iv) This question was well answered with most candidates understanding what was being asked and identifying that the number of red balls could be 0, 1 or 2. Few candidates included 3 in their table as there were three red balls in the box. Stronger responses identified that two of the required probabilities were the answers to **part (i)**, P(2) and **part (ii)** P(1) and that P(0) was 1 – ‘the sum of the other two probabilities’.

(v) Most candidates correctly used the formula for the Variance of X with only a few omitting to square the E(X). Better responses showed all their working clearly for both the E(X) and the Variance(X) even though it was possible to evaluate E(X) and E (X^2) using mental arithmetic.

Question 7

(i)(a) This question was well answered. Almost all candidates recognised it as a Binomial probability question with n = 6 and p = 0.3. A common error made by candidates was to omit the third term concerning two people being Notes supporters. Other candidates did not appear to understand the ‘success/fail’ requirement of a binomial and attempted a calculation with 0.3 and 0.45. Very few candidates did not gain the final accuracy mark, either due to premature approximation or giving the answer to just two significant figures.

(b) Few candidates were successful in answering this question. Many candidates instead gave the probability of all 6 people by using 0.75^6, rather than none of the 6.

(ii) This question was very well answered. Most candidates identified the need for a normal approximation and correctly calculated the mean and variance from the values n = 240 and p = 0.25. Some candidates used their incorrect answer to **part (i)(b)** which was often p = 0.178. Several candidates correctly substituted their mean and standard deviation into a standardisation formula and included the continuity correction and went on to find the appropriate area for their final answer.

MATHEMATICS

Paper 9709/72
Paper 72

Key messages

If candidates use the additional page or additional sheets for working, the question number must be clearly indicated.

It is important that candidates round answers to the number of significant figures as required in the question. Standard statistical notation must be used correctly for this component.

It is important that candidates understand how to recognise an underlying distribution and can also apply and justify a suitable approximating distribution when required to do so.

When carrying out a significance test the comparison between the test value and critical value, or equivalent, must be clearly shown in order to justify the conclusion. When making a conclusion to a significance test it should be in context and not definite.

General comments

There were some very good responses to this paper, however it was apparent that some candidates were not fully prepared for the demands of the examination. **Question 4**, in particular **parts (i)** and **(ii)**, were well attempted as was **Question 5**, whilst **Question 6** proved to be more challenging for candidates.

It is important that candidates can round correctly to 3 significant figures. There were occasions where only 2 significant figures were given with no indication of more accurate figures; this would result in not gaining accuracy marks.

There did not appear to be any time issues for candidates on this paper, and responses were generally well presented.

Comments on specific questions

Question 1

In this question, some candidates did not appear to understand the difference between the actual distribution of X and an approximating distribution. The given scenario was Binomial, $\mathrm{B}\left(500, \frac{1}{150}\right)$, which, because the relevant conditions were satisfied, could then be approximated to a Poisson distribution, namely $\mathrm{P}\left(\frac{10}{3}\right)$. The calculation in **part (iii)** should therefore have been done using a Poisson distribution. Many candidates used an incorrect Normal distribution in **part (iii)** or did not use an approximating distribution at all and found the probability using their Binomial distribution. Whilst some credit could be gained for this, it was not what was required by the question. It is important that candidates understand how to recognise an underlying distribution and if requested, as in this question, can then apply and justify an approximating distribution. When justifying an approximating distribution merely stating, for example, $np < 5$ is not sufficient; to fully justify the approximation it must be clear what np is equal to in the context given, so here it needs to be clear that $np = \frac{10}{3}$ and is less than 5. Some candidates approximating prematurely and using 3 or 3.3 for λ rather than $\frac{10}{3}$, therefore were unable to gain the relevant accuracy marks.

Question 2

In **part (i)(a)** many candidates omitted to give an assumption, or gave an incorrect one. The calculation to find *n* was generally well attempted, though some candidates did not realise that *n* should be an integer. Common errors included a sign error when setting up the initial equation and algebraic errors in rearranging this equation.

The null and alternative hypotheses should have been clearly stated when carrying out the test, and it was important here, in **part (i)(b)**, that the comparison between the test value and the critical value was clearly shown; this could be either as an inequality statement (e.g. $-2.182 < -1.751$) or on a clearly labelled diagram. Some diagrams drawn by candidates were not fully labelled and suggested, rather than stated, that –2.182 was in the rejection region without a full explanation of where the rejection region was. Some candidates recalculated the test statistic –2.182, often incorrectly, with many using their own incorrect figures from **part (i)**. It is important that the final conclusion to the test should be in context and not definite.

Few candidates gave a fully correct answer to **part (ii)**. It was important that the distribution that was identified as 'unknown' or 'not Normal' was clearly the Population. Responses such as 'It is not Normal' or 'the distribution is unknown' were not acceptable.

Question 3

Part (i) was reasonably well attempted. A minority of candidates did not calculate the unbiased estimate for the population variance and used the biased variance instead. There was confusion between the two formulae for the unbiased variance. Other errors included use of incorrect z values and not writing the answer as an interval.

Some candidates gave good answers for **part (ii)**, but many, although realising that the interval was wider, were unable to give a convincing reason. Some candidates thought the interval would be narrower.

Question 4

Parts (i) and **(ii)** of this question were well answered. In **part (i)** the most common method used was to integrate f(x) and equate to 1. In **part (ii)** most candidates successfully integrated xf(x) using correct limits and thus reached the required answer. **Part (iii)** was more challenging for candidates. The most common method was firstly to find the median by integrating f(x) and equating to 0.5, where many candidates successful found that the median was $\sqrt{2}$ and then integrate between E(X) and their median to find the required probability. Many candidates tried to integrate f(x) but used incorrect limits. The more concise method, without finding the actual value of the median, was to find the probability of less than E(X) then calculate 0.5 minus this probability; this method was rarely seen.

Question 5

Most candidates identified what was required in this question, although, on both parts of the question, some made errors when finding the variance. Other candidates, whilst finding the correct value for z were then unable to find the correct probability. Occasionally candidates made errors by confusing variance and standard deviation. In **part (ii)** some candidates did not achieve the required accuracy by rounding to 3 decimal places rather than the required 3 significant figures; if more accurate figures were not seen before rounding the candidate could not gain an accuracy mark.

Question 6

In **part (i)** the steps required, after stating the Hypotheses, were to calculate $P(X \geqslant 4)$ and $P(X \geqslant 5)$ and show that the first was >0.01 and the second <0.01. Many candidates found only the second of these probabilities, and some candidates did not answer to the 3 significant figure accuracy required. Other candidates, when attempting to find $P(X \geqslant 5)$ merely found $P(X = 5) + P(X = 6)$. Many candidates unnecessarily calculated $P(X \geqslant 6)$ in order to reach their conclusion.

In **part (ii)** it was important that the explanation of a Type I error was not a textbook definition but was put into the context of the question. Few candidates successfully found its probability.

Part (iii) was reasonably well attempted with some candidates identifying that they should calculate $P(X \geqslant 4)$ with $\lambda = 7.0$

MATHEMATICS

Paper 9709/73
Paper 73

Key messages

Candidates were required to state assumptions for two of the questions; these were often omitted or incorrect versions were suggested.

Conclusions to significance tests should be written in the context of the question, along with the assumption for **Question 4** needed to be written in the context of the question.

General comments

Many candidates' presentation of their work was clear and well explained.

The significance test in **Question 4** which involved the Poisson distribution required candidates to consider the tail of the distribution.

Comments on specific questions

Question 1

(i) Many correct answers were seen to this part. Some candidates gave the variance as 49.6.

(ii) Many correct answers were seen to this part. A variety of attempts at the variance were seen in candidate responses, with the most common incorrect values being 46.6, 52.6, 12.4, 15.4 and 9.4.

Question 2

(i) The majority of candidates answered this question correctly, changing to the new Poisson parameter (4.8) and calculating the relevant 4 terms.

(ii) Many candidates correctly used a normal distribution as their suitable approximating distribution. The appropriate normal distribution was N(144,144). With this change, a continuity correction factor was required (139.5). Some candidates omitted the continuity correction factor and some candidates used 140.5. It was necessary to select the larger area for the probability. A diagram could help with both the choice of the continuity correction factor and the selection of the correct probability.

Question 3

(i) The question required that a necessary assumption be stated. This was to assume that the population was normally distributed. Many other incorrect or incomplete suggestions were made, including 'the sample was normally distributed', 'the distribution was normally distributed' and 'this data was normally distributed'. Another frequently seen response was to state that 'the standard deviation was the same'; this was given and was not an assumption.

To find the confidence interval, the sample mean (25.9) and the value of z (2.17), both of which in the correct form, were required. The standard deviation was given and was required to be used, rather than an estimate from the sample.

Many candidates used these values correctly and gave an interval for their answer.

(ii) For neither of the two extra intervals to contain the true value of μ, the product of the probabilities (0.03 × 0.03) was required; some candidates found this product. Other candidates suggested answers of 0.03 or 0.06 or 0.03^3 or 0.97^2.

Question 4

This question required that an assumption be stated, this was to assume that the trains ran independently. It was appropriate that the context of trains or times was included in this statement. The appropriate distribution was the binomial distribution B(20, 0.92) and the significance test required the hypotheses H_0 : P(on time) = 0.92 and H_1: P(on time) < 0.92.

For the test with the binomial it was necessary to calculate the probability of the tail (0.0706) for P($\leqslant$ 16) and then to write down the comparison with 0.05 or to have used a similar approach. The conclusion should then have been expressed in the context of the question with suitable non-definite wording. Some candidates followed this procedure very well. Other candidates used a tail of P($\leqslant$ 15) which gave the reverse conclusion. Other candidates incorrectly used the single value P(16) instead of a tail.

Although the parameters involved in this question led to the binomial distribution, some candidates tried to use a normal distribution with either integral values or with proportions. A Special Case in the mark scheme allowed such candidates to score 2 marks provided this work was carried through correctly.

Question 5

(i) The majority of candidates answered this question correctly, changing to the new Poisson distribution $P_o(3)$ and calculating the relevant 3 terms. Some candidates tried an incorrect approach by finding the product of P(0 – 6) and P(0 – 2).

(ii) Many candidates made a correct start to this question and solved the equation correctly. Some candidates made algebraic errors. Other candidates incorrectly gave ± 1.41 as the answer.

(iii)
(a) Many candidates wrote down a correct inequality. Some candidates started to simplify this in **part (a)**; this work could be used in **part (b)**.

(b) Many different attempts to solve the inequality were seen. Some candidates used algebra but made errors such as simplifying $\frac{(n+1)!}{n!}$ to n instead of (*n* + 1). Some candidates attempted to use logs, however candidates taking this approach were often unsuccessful. Other candidates attempted a numerical approach, this was acceptable provided that sufficient calculations were shown to establish the result for *n*.

Question 6

(i) Many correct solutions for this question were seen, the majority of candidates integrated correctly and equated the result to 1 which was essential in correctly answering this question.

(ii) The majority of candidates integrated correctly and substituted the correct limits to find the probability.

(iii) Many correct answers were seen. A common error was for candidates not square E(X) when subtracting. Several candidates found only $\frac{2}{9}\int_0^3 (3x^3 - x^4)\,dx$ and did not continue further than this.

Question 7

(i) This part of **Question 7** was well answered by many candidates. The correct hypotheses were stated and the normal distribution of the means of weekly incomes was used. The comparison of the test statistic with the critical value for the one-tailed test (1.845 < 1.96) or the probability version was correctly shown. The conclusion was stated in context and in non-definite form.

Some candidates incorrectly used '*x*' or just income or no symbol in the hypotheses. Some candidates used the wrong value for *z* such as 2.24, whilst several other candidates gave the incorrect reverse conclusion. Other candidates used the critical value method (583.19); this work could be re-used in **part (ii)**.

(ii) Several candidates were able to follow the steps required to find the probability of the Type II error correctly. Some of these candidates followed the method correctly but had a slight inaccuracy in their answer, this was due to premature approximation caused by using 583 in their working instead of 583.19.

Many candidates did not find the critical value for the acceptance region, and instead used a different value and were able to score method marks for subsequent work.