Scheme of Work

Cambridge International AS & A Level

Cambridge International AS & A Level
Mathematics

9709/02 Pure Mathematics 2 (P2)
For examination from 2017

Cambridge Advanced

CAMBRIDGE
International Examinations

Contents

Introduction

This scheme of work provides ideas about how to construct and deliver a course. It has been broken down into different units of the three subject areas of Pure Mathematics (units P1, P2 and P3), Mechanics (units M1 and M2) and Probability & Statistics (units S1 and S2). For each unit there are suggested teaching activities and learning resources to use in the classroom for all of the syllabus learning objectives.

This scheme of work, like any other, is meant to be a guideline, offering advice, tips and ideas. It can never be complete but hopefully provides teachers with a basis to plan their lessons. It covers the minimum required for the Cambridge International AS & A Level course but also adds enhancement and development ideas. It does not take into account that different schools take different amounts of time to cover the Cambridge International AS & A Level course.

The mathematical content of Pure Mathematics 2 in the syllabus is detailed in the tables below. The order in which topics are listed is not intended to imply anything about the order in which they might be taught.

Recommended prior knowledge

Knowledge of the content of unit P1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions.

Candidates will be expected to be familiar with scientific notation for the expression of compound units, e.g. 5 m s^{-1} for 5 metres per second.

As well as demonstrating skill in the appropriate techniques, candidates will be expected to apply their knowledge in the solution of problems. Individual questions set may involve ideas and methods from more than one section of the relevant content list.

Outline

Suggestions for independent study **(I)** and formative assessment **(F)** are indicated, where appropriate, within this scheme of work. The activities in the scheme of work are only suggestions and there are many other useful activities to be found in the materials referred to in the learning resource list.

Opportunities for differentiation are indicated as **basic/consolidation** and **challenging/extension**. There is the potential for differentiation by resource, length, grouping, expected level of outcome, and degree of support by the teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgment of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

Teacher support

Teacher Support (http://teachers.cie.org.uk) is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online.

This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on Teacher Support at http://teachers.cie.org.uk. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org.

Resources

The up-to-date resource list for this syllabus, including textbooks endorsed by Cambridge, is listed at www.cie.org.uk

Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. As such, all textbooks endorsed by Cambridge for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective.

Websites and videos

This scheme of work includes website links providing direct access to internet resources. Cambridge International Examinations is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services).

The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

1. Algebra

Learning objectives	Suggested teaching activities
Understand the meaning of $\|x\|$ and use relations such as $\|a\| = \|b\| \Leftrightarrow a^2 = b^2$ and $\|x - a\| < b \Leftrightarrow a - b < x < a + b$ in the course of solving equations and inequalities.	To introduce the notation, start with a numerical value, e.g. –5, and discuss the meaning of $\|-5\|$. You could help learners to deduce the results $\|a\| = \|b\| \Leftrightarrow a^2 = b^2$ and $\|x - a\| < b \Leftrightarrow a - b < x < a + b$ as part of a class discussion. This link leads to four files which are extremely useful. Click on https://www.tes.co.uk/teaching-resource/a-level-maths-c2-modulus-function-worksheets-6146818 and log in for free download. ‘Modulus Function Introduction’ provides a worksheet for learners to complete. **(I)** ‘Solving Modulus Equations and Inequalities’ could be used for consolidation/practice. **(I)** ‘Modulus Transformations’ provides practice at sketching graphs involving a modulus. You could demonstrate some initially to learners using a graph plotter. **(I)** ‘Alternative Methods for Solving Modulus Equations’ is a worksheet which helps learners to explore the different ways of solving this type of equation. **(I)** The link below demonstrates the graphs of various modulus functions. http://www.mathsmutt.co.uk/files/mod.htm **Past papers: (I)(F)** June 2014 paper 21, question 1 June 2014 paper 22, question 1 November 2014 paper 22, question 1 June 2013 paper 21, question 1 (involves logarithms) June 2013 paper 22, question 2
Divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero).	There are several different methods of polynomial division including inspection, the table method, and long division. This PowerPoint presentation introduces all three methods for factorising cubics. You can use the methods for any polynomial and also for division that results in a remainder: http://www.furthermaths.org.uk/files/sample/files/edx/Factorising_cubics.ppt When teaching any of the methods, start with a numerical example to remind learners of the thought process they need, and use this to introduce the terms ‘quotient’ and ‘remainder’ . For example $54763 \div 8$ leads to a quotient of 6845 and a remainder of 3. Continue with a simple algebraic example $(x^2 + 4x + 1) \div (x + 2)$ which leads to a

Learning objectives	Suggested teaching activities
	quotient of $x+2$ and a remainder of -3. You will probably need to show learners further examples involving more complex polynomials before they practise on their own. The links below provide ideas on possible approaches you can take for long division: https://www.khanacademy.org/math/algebra2/polynomial_and_rational/dividing_polynomials/v/dividing-polynomials-with-remainders https://www.mathsisfun.com/algebra/polynomials-division-long.html This link has a work sheet of examples for practising any of the methods for division. http://www.mathworksheetsgo.com/sheets/algebra-2/polynomials/dividing-polynomials-worksheet.php **(I)** There is another approach known as synthetic division but learners have to be careful when using it, especially when factorising. You will find many useful questions in textbooks for learners to practise. **Past papers: (I)(F)** June 2014 paper 21 question 3
Use the factor theorem and the remainder theorem, e.g. to find factors, solve polynomial equations or evaluate unknown coefficients.	Summarise the work already done on polynomial division to show that $p(x)$ = (divisor $\times$ quotient) + remainder. Show that algebraic division can often be avoided in questions by substituting into $p(x)$ the value of x that makes the divisor zero (e.g. substituting 3 if the divisor is $x - 3$ and calculating p(3) to find the remainder). Show that the factor theorem is a special case of the remainder theorem when the remainder is zero. The link below gives a good approach of this type which you could use with a whole class. https://www.mathsisfun.com/algebra/polynomials-remainder-factor.html You could show examples involving finding factors, solving polynomial equations and evaluating unknown coefficients to the whole class, questioning learners individually throughout. Remind learners that they should show all their working as the use of a calculator for finding solutions to polynomial equations will not be accepted in an exam. Here is a useful worksheet which covers basic use of the remainder theorem and evaluating unknown coefficients (log in for free download): https://www.tes.co.uk/teaching-resource/worksheet-on-the-remainder-theorem-6140286 **(I)**

Learning objectives	Suggested teaching activities
	This link gives more examples on the remainder theorem and on solving polynomial equations. http://www.mash.dept.shef.ac.uk/Resources/A26remainder.pdf **(I)** **Past papers: (I)(F)** June 2014 paper 21, question 3(ii) June 2014 paper 22, question 6 November 2014 paper 21, question 5 June 2013 paper 21, question 4 June 2013 paper 22, question 3

2. Logarithmic and exponential functions

Learning objectives	Suggested teaching activities
Understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base).	Start by defining the terms 'logarithm' and 'exponential', linking to the concept of indices. To help learners understand a statement such as $\log_a x = b$, you could describe it to them in words such as "What power of *a* is *x*? Answer: *b*" This link gives an introduction with animation showing the relationship between logarithms and exponentials http://www.purplemath.com/modules/logs.htm . Learners should practise converting expressions from logarithmic to exponential form and from exponential form to logarithmic. Most textbooks will have plenty of examples of this type. A useful worksheet is here (includes the laws of logarithms). http://maths.mq.edu.au/numeracy/web_mums/module2/Worksheet27/module2.pdf **(I)** To introduce the laws of logarithms, start with statements $\log_a x = b$ and $\log_a y = c$. Use targeted questioning to encourage learners to write the exponential forms of these statements and reach the conclusion that $a^{b+c} = xy$, rewriting this in logarithmic form to obtain $\log_a xy = \log_a x + \log_a y.$ You could ask learners to obtain the other two laws in a similar way. Learners will then need to practise applying these laws. The link below provides eight files of notes, worksheets and revision (log in for free download). https://www.tes.co.uk/teaching-resource/a-level-maths-logarithms-worksheets-and-revision-6146791 **(I)** The link below provides an additional resource which demonstrates the above approach. http://www.mathsisfun.com/algebra/exponents-logarithms.html **Past papers: (I)(F)** November 2014 paper 21, question 5(ii) June 2013 paper 21, question 2
Understand the definition and properties of e^x and In *x*, including their relationship as inverse functions and their graphs.	You could introduce the exponential function e^x in various ways. One approach would be using a graph plotter to show learners the graphs of various exponential functions e.g. $y = 2^x$, $y = 3^x$, $y = 5^x$. Develop the idea of a particular exponential function which lies between

Learning objectives	Suggested teaching activities
	$y=2^x$ and $y=3^x$, such that its gradient function is the same as itself. With a suitable graph plotter you could demonstrate that the gradient function of $y=e^x$ is e^x. There are other, formal, approaches that you could use with more capable learners. For example you could consider compound interest and the limit of the series $\left(1+\frac{1}{n}\right)^n$ as shown at this link: http://www.mathsisfun.com/numbers/e-eulers-number.html You could encourage learners to obtain the logarithmic form of the statement $e^x=a$ and so introduce them to natural logarithms. Building on the work done on functions in unit P1, you could develop this into the inverse relationship between e^x and $\ln x$ and demonstrate the inverses on a graph plotter. The link below leads to an interactive exercise covering this relationship: http://hotmath.com/help/gt/genericalg2/section_8_5.html **(I)**
Use logarithms to solve equations of the form $a^x=b$, and similar Inequalities.	As a whole class exercise, you could work through some examples of increasing difficulty, using carefully directed questioning to work through the solutions. Textbooks will include many examples of this type of question and the interactive exercise at the link above includes some too. You could demonstrate examples using inequalities, with learners finding critical values first and then deducing the set of solutions. It is helpful to highlight to learners the sign of ln *x* for $0<x\le 1$, perhaps through an example where the inequality reverses. **Past papers: (I)(F)** November 2014 paper 22, question 4 June 2013 paper 21, question 1
Use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.	If you can relate this technique to practical situations, this could help learners when they need to use it in their scientific subjects. Common forms of equation are $y=Ab^x$ and $y=Ax^b$. Learners will need to be able to write these equations in logarithmic form and hence relate them to the equation of a straight line. Sometimes the

Learning objectives	Suggested teaching activities
	variables will be letters other than *x* and *y* so learners need to spot the form of the equation in order to distinguish the variables from the constants. This link provides a useful summary for dealing with situations involving $y = Ax^b$ http://mathbench.umd.edu/modules/misc_scaling/page11.htm You could work through this with learners in class or they could study it independently. **(I)** You could use a similar approach for equations of the type $y = Ab^x$. Work through such an example in class, making use of a graph plotter to demonstrate the straight line obtained**.** Textbooks will provide learners with many useful practice questions. For variety, try to choose examples which involve variables other than *x* and *y*. Often, learners are asked to work from a given graph in straight line form. Common errors involve learners considering y values rather than $\ln y$ values, so they will need to practice questions to avoid such errors. The P2 past exam papers have examples of this type. To help reinforce this point, you could split the learners into groups or pairs and ask each of them to prepare a question. The easiest way would be for them to 'work backwards' from a logarithmic relationship e.g. $P = At^b$. Each group could choose values for A and b, work out the coordinates of two pairs of coordinates and draw an appropriate straight line graph. Learners could circulate their graphs around the other groups who would then identify the logarithmic equations used to draw the graphs**.** **Past papers: (I)(F)** June 2014 paper 22, question 5 November 2014 paper 21, question 2 June 2013 paper 22, question 4

3. Trigonometry

Learning objectives	Suggested teaching activities
Understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude.	You could start by defining the secant, cosecant and cotangent functions. Learners should know the graphs of the sine, cosine and tangent functions so, as a group or individual task, you could ask them to think what the graphs of the secant, cosecant and cotangent functions would look like. For instance, you could give them the graph of *y* = sin *x* (from -360° to 720°) and ask them to sketch *y* = cosec *x* on the same axes. Then they could check using a graph plotter. A similar graphical approach could be used for *y* = sec *x* and *y* = cot *x*.
Use trigonometrical identities for the simplification and exact evaluation of expressions and, in the course of solving equations, select an identity or identities appropriate to the context, showing familiarity in particular with the use of: - $\sec^2\theta \equiv 1+\tan^2\theta$ and $\text{cosec}^2\theta \equiv 1+\cot^2\theta$ - the expansions of sin(*A* ± *B*), cos(*A* ± *B*) and tan(*A* ± *B*) - the formulae for sin 2*A*, cos 2*A* and tan 2*A* - the expressions of $a\sin\theta + b\cos\theta$ in the forms $R\sin(\theta\pm\alpha)$ and $R\cos(\theta\pm\alpha)$.	You could start with the identity $\sin^2\theta+\cos^2\theta\equiv 1$ (which learners know already) and ask what they find when (a) they divide each term in this identity by $\cos^2\theta$ and (b) they divide each term in the original identity by $\sin^2\theta$. The link below provides two files, one of which is a matching exercise and the other a worksheet for learners to complete as consolidation and practice (log in for free download): https://www.tes.com/teaching-resource/a-level-maths-reciprocal-trig-functions-worksheet-6146865 **(I)** Learners will need plenty of practice at simplifying trigonometric expressions and using the identities, particularly questions of the 'Show that' or 'Prove that' type. The best strategy is to start with one side of the expression (usually the left hand side) and manipulate it using the identities covered so far. Textbooks will include some practice questions. This link provides an exercise on simplification. http://worksheets.tutorvista.com/proving-trigonometric-identities-worksheet.html **(I)** This link provides an exercise on proof. https://people.math.osu.edu/maharry.1/150Au2011/TrigIdentities.pdf **(I)** Learners will need to be able to use the identities to solve equations in degrees or radians, and textbooks will contain useful exercises on this. Learners will also need to practise manipulating expressions to obtain an equation (usually quadratic) in terms of one trigonometric ratio e.g. $2\sec^2\theta-3+\tan\theta=0$ will simplify to $2\tan^2\theta+\tan\theta-1=0$ which factorises. For the compound angle (addition) formulae, it is a good idea to work through an example of how one formula is

Learning objectives	Suggested teaching activities
	derived, perhaps as a whole class exercise. A video proof is here https://www.youtube.com/watch?v=a0LvqflQMx4 The link below covers the proof of one formula in a similar way. As an exercise for more capable learners, you could ask them to work out the proofs of some of the other formulae. http://www.trans4mind.com/personal_development/mathematics/trigonometry/compoundAngleProofs.htm#mozTocId169602 Alternatively, you could start by giving learners the challenge of deriving the compound angle formulae graphically using this interesting investigation: https://www.tes.co.uk/teaching-resource/the-compound-angle-formulae-lesson-worksheet-6056103 Proving the formulae may then come more easily to learners once they are more familiar with them. Once learners are competent with the compound angle formulae, you could ask them to derive the double angle formulae. They will need to find all possible variants of the formula for $\cos 2\theta$ as well as rearranging them to $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ and $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ for use in other applications such as integration. Textbooks include many useful practice exercises on solving equations using the compound and double angle formulae. You should ensure that learners are proficient at using radians as well as degrees. **(I)** This link gives a clear summary of how to deal with expressions of the type $a\sin\theta + b\cos\theta$ http://www.intmath.com/analytic-trigonometry/6-express-sin-sum-angles.php Start with an example e.g. $3\sin\theta + 4\cos\theta$ and show that it may be written in the form $5\sin(\theta + 53.13^{\circ})$. This could also be verified by use of a graph plotter: show learners the graph of $y = 3\sin\theta + 4\cos\theta$ and, with a discussion on transformations, you could encourage learners to write this expression in a different way. They could check the result by plotting the equivalent expression and seeing that it gives the same graph. Ask learners to find the maximum and minimum values of the expression and the values of θ at which they occur. (You should discourage the use of calculus for questions of this type.) Textbooks include many examples of writing equivalent expressions, solving equations and finding maximum and minimum values. Learners need to be proficient at using radians as well as degrees. **(I)**

Learning objectives	Suggested teaching activities
	Past papers: (I)(F) June 2014 paper 21, question 5(ii)(a) June 2014 paper 22, question 2 November 2014 paper 21, question 7 November 2014 paper 22 question 7 June 2013 paper 21, question 3 and question 8 June 2013 paper 22, question 8

4. Differentiation

Learning objectives	Suggested teaching activities
Use the derivatives of e^x, ln *x*, sin *x*, cos *x*, tan *x*, together with constant multiples, sums, differences and composites.	It is probably best to teach this section using a whole class approach and targeted questioning of learners. For the function $y = e^x$, learners already know that the gradient function is e^x so you can build on this by differentiating other functions such as $y = e^{mx}$, $y = e^{f(x)}$, making use of the chain rule where appropriate. To differentiate $y = \ln x$, write $x = e^y$, so $\frac{dx}{dy} = e^y$ and you can obtain the result $\frac{dy}{dx} = \frac{1}{x}$. Using the chain rule, you can generalise to expressions of the form $\frac{d}{dx}\left(\ln f(x)\right) = \frac{f'(x)}{f(x)}$. Textbooks will have exercises for learners to practice. **(I)** To obtain the derivatives of $\sin x$ and $\cos x$, you could consider the gradient of a chord from the origin to a point (*h*, sin *h*) on the curve *y* = sin *x*. Ask learners to calculate the gradient sin *h* / *h* (where *h* is 0.1 then 0.01 then 0.001) and use this to deduce the gradient at *x* = 0. They can deduce the gradient at other key points on the graph, for instance *x* = 0, *π*/2, *π, 3π*/2, 2*π*, use their values to plot the gradient function on a graph of *y* = sin *x* and name the graph obtained. Show them that a similar approach will give them the gradient function for *y* = cos *x*. You can find this method in many textbooks. It is also covered at the link below, together with differentiation from first principles which is suitable as an extension for the more capable learner: http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-sincos-2009-1.pdf You could encourage learners to obtain results for the derivatives of sin *mx*, cos *mx*, sin f(*x*) and cos f(*x*) during a class discussion, making use of the chain rule. Leave the differentiation of *y* = tan *x* until the quotient rule has been covered. Many textbooks will have exercises for learners to practice. **(I)** **Past papers: (I)(F)** June 2014 paper 21, question 2 June 2014 paper 22, question 8

Learning objectives	Suggested teaching activities
	November 2014 paper 21, question 4
Differentiate products and quotients.	It would be an advantage to derive the product and quotient rules as a whole class exercise so that learners (especially the more capable) can understand the formulae more thoroughly. There is a proof here using function notation http://nrich.maths.org/10086. Alternatively, you can write the product as *uv* (where *u* and *v* are functions of *x*) then consider increasing the area of a rectangle *uv* to ($u + \delta u$)($v + \delta v$). Expanding the brackets, writing every term over δx and considering the limit as δx -> 0 leads to the product rule. The link below leads to three files of examples and worksheets on differentiation of products (log in for free download): https://www.tes.co.uk/teaching-resource/product-and-quotient-rules-6146838 **(I)** Appropriate textbooks will have further examples. Try to introduce a variety of different types of functions (such as those in the previous section) and encourage learners to simplify their answers. You could set learners the task of deriving the quotient rule by differentiating $y = \frac{u}{v}$, where u and v are functions of x, as a product $y = uv^{-1}$, using the product rule. Ask learners to differentiate $y = \tan x$ using the quotient rule. The link below gives three files which include examples/worksheets on differentiation of quotients (log in for free download): https://www.tes.co.uk/teaching-resource/product-and-quotient-rules-6146838 **(I)** Appropriate textbooks will have further examples. Try to introduce a variety of different types of functions (such as those in the previous section) and encourage learners to simplify their answers. **Past papers: (I)(F)** June 2014 paper 21, question 2 November 2014 paper 21, question 6(ii) June 2013 paper 22, question 7(b)
Find and use the first derivative of a function which is defined parametrically or implicitly.	You can introduce the idea of parametric equations to learners by asking them to imagine two cars moving towards each other along different straight lines on the *x*-*y* plane. You know their lines will intersect but how do

Learning objectives	Suggested teaching activities
	you know if the cars will collide or miss each other? You need to consider a third parameter (e.g. time), and express both *x* and *y* in terms of this parameter, in order to say whether or not there will be a collision. Then you can show learners some simple examples e.g. $x = 2t,\ y = 3t^2 + 5$ and eliminate *t* to obtain the Cartesian form of the curve. A graph plotter may be useful. Show that the gradient function may be obtained using the derivatives $\frac{dx}{dt}$ and $\frac{dy}{dt}$ together with the chain rule. Extend the work to include parametric equations involving trigonometric functions e.g. $y = 3\cos 2\theta,\ x = 4\sin\theta$ to help learners to consolidate their knowledge of trigonometric identities and differentiation of trigonometric functions. This link gives a clear and thorough treatment of the topic with worked examples. See 17.1 Cartesian and parametric equations and 17.4 Parametric differentiation: http://www.cimt.plymouth.ac.uk/projects/mepres/alevel/pure_ch17.pdf The link below provides a good overview of the topic (second derivatives are not required) http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-parametric-2009-1.pdf As an extension, learners could investigate interesting curves expressed in parametric form using a graph plotter. There are many websites with good examples, for instance this one gives a selection of equations. https://cims.nyu.edu/~kiryl/Calculus/Section_9.1--Parametric_Curves/Parametric_Curves.pdf For implicit differentiation, start with the definition of implicit and explicit functions. Ask learners to consider e.g. $y^2 = x$, re-write it as $y = x^{\frac{1}{2}}$ then differentiate to obtain $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$. They can re-write this as $\frac{dy}{dx} = \frac{1}{2y}$ leading to the statement $2y\frac{dy}{dx} = 1$. Repeat this exercise with several similar examples (powers of *y*) so that learners can identify a pattern. This exercise could be done with the whole class or with groups. Show learners terms of various types: they now know how to differentiate powers of *x* or *y* with respect to *x*. You can introduce the idea of a product term by asking them to differentiate equations such as $xy = x$, $x^2y^3 = 4$

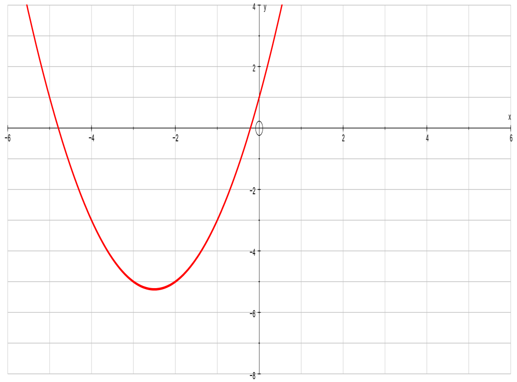
Learning objectives	Suggested teaching activities
	implicitly using the product rule and by rearranging them and differentiating y with respect to x. You can now ask learners to work through an equation from left to right and differentiate it implicitly without rearranging it first. (It is a good idea to give them equations which cannot be rearranged to prevent this.) The links below provide useful examples or worksheets: https://www.khanacademy.org/math/differential-calculus/taking-derivatives/implicit_differentiation/v/implicit-derivative-of-x-y-2-x-y-1 http://www.intmath.com/differentiation/8-derivative-implicit-function.php http://cdn.kutasoftware.com/Worksheets/Calc/03%20-%20Implicit%20Differentiation.pdf **Past papers: (I)(F)** June 2014 paper 21, question 7 June 2014 paper 22, question 4 November 2014, paper 21, question 4(ii) November 2014 paper 22, question 3 June 2013 paper 21, question 5 June 2013 paper 22, question 5

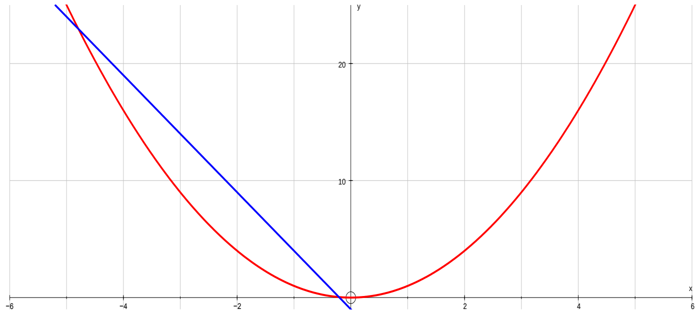
5. Integration

Learning objectives	Suggested teaching activities
Extend the idea of 'reverse differentiation' to include the integration of e^{ax+b}, $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$ and $\sec^2(ax+b)$ (knowledge of the general method of integration by substitution is not required).	Start with a quick review of integration from unit P1, perhaps as a question and answer session with learners writing on mini whiteboards and holding up their responses. This will enable you to assess all learners' understanding before moving on to examples in this section. You could divide learners into groups and give them sets of expressions to integrate. Ask them to consider what would need to be differentiated to obtain the given expression, then to work out some general principles. The following link provides a good approach for integration involving logarithmic functions. Some of the examples may be beyond the range of this syllabus. http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-inttologs-2009-1.pdf **(I)** Textbooks will include exercises on integrating all of these types of function, including finding areas. **(I)** **Past papers: (I)(F)** June 2014 paper 21, question 5 June 2014, paper 21, question 6(a) June 2014 paper 22, question 3(a) June 2014 paper 22, question 7 November 2014 paper 21, question 3(b) November 2014 paper 22, question 2 November 2014 paper 22, question 5 June 2013 paper 22, question 1 June 2013, paper 22, question 7(a)
Use trigonometrical relationships (such as double-angle formulae) to facilitate the integration of functions such as $\cos^2 x$.	Ask learners to recall the three forms of the trigonometric identity for $\cos 2x$ and then to use them to rewrite $\cos^2 x$ and $\sin^2 x$ in terms of $\cos 2x$. Introduce learners to integrals of the type $\int 2\sin x\cos x\,\mathrm{d}x$, $\int \cos^2 2x\,\mathrm{d}x$ and $\int \tan^2 3x+1\,\mathrm{d}x$. Appropriate textbooks will have examples of these. Try to relate them to areas and also to simple first order differential equations, for example: find the equation of the curve, with gradient function $\frac{\mathrm{d}y}{\mathrm{d}x}=2\sec^2 x+1$ for

Learning objectives	Suggested teaching activities
	$0 \le x < \frac{\pi}{2}$, which passes through the point $x = \frac{\pi}{4}$ **(I)** **Past papers: (I)(F)** November 2014, paper 21, question 3(a) June 2013 paper 21, question 3
Use the trapezium rule to estimate the value of a definite integral, and use sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or an under-estimate.	You could start by sketching on the board part of a curve with an unknown equation. Ask learners to consider the area under this curve, enclosed by the *x*-axis, split into a number of strips of equal width. How could they work out the area? This link leads to a PowerPoint presentation which uses this approach and gives some examples (log in for free download): https://www.tes.co.uk/teaching-resource/trapezium-rule-powerpoint-c2-maths-lesson-3009786 Encourage learners to determine, from a sketch of the curve, whether or not the area they calculate will be an overestimate or underestimate. It is important for them to be able to explain their reasoning clearly. The link below leads to three files which would enable learners to work through examples on the trapezium rule and understand its limitations. It includes clear explanations of overestimates and underestimates https://www.tes.co.uk/teaching-resource/trapezium-rule-6146799 **(I)** **Past papers: (I)(F)** June 2014 paper 21, question 6(b) June 2014 paper 22, question 3(b) November 2014 paper 21, question 1

6. Numerical solution of equations

Learning objectives	Suggested teaching activities
Locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change.	You could introduce this topic by using a graph plotter to demonstrate both sign changes and graphical considerations e.g. $y = x^2 + 5x + 1$: Change of sign  In this way, you can see clearly that there are solutions to the equation $x^2 + 5x + 1 = 0$ in the intervals $-5 < x < -4$ and $-1 < x < 0$. Learners consider the sign of y either side of the points of intersection of the curve with the x -axis i.e. using the boundaries above. Demonstrate also that the same result may be obtained by plotting $y = x^2$ against $y = -5x - 1$.

Learning objectives	Suggested teaching activities
	 Learners will need to practise examples of both types. You should encourage them to set out their work clearly and accurately. For example, to show that the equation $x^2 = -5x - 1$ has a solution in the interval $-5 < x < -4$, learners should state 'Let $f(x) = x^2 + 5x + 1$' then write the equation as $f(x) = 0$. By calculating and writing down the values of $f(-5)$ and $f(-4)$, they can demonstrate that there is a sign change and state their conclusion e.g. ' There is a change of sign, so a solution lies in the interval $-5 < x < -4$'. This link includes a useful overview of the topic, with examples: http://www.cimt.plymouth.ac.uk/projects/mepres/alevel/pure_ch19.pdf
Understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation.	The second part of this chapter deals with convergence to a root of an equation. http://www.cimt.plymouth.ac.uk/projects/mepres/alevel/pure_ch19.pdf The first part of this link demonstrates a formal approach to the idea of a sequence of approximations converging to a root of an equation. You could use it with able learners or perhaps with a whole class. It explains how an iterative formula generates the sequence; this is the next learning objective. http://www-solar.mcs.st-andrews.ac.uk/~clare/Lectures/num-analysis/Numan_chap2.pdf
Understand how a given simple iterative formula of the form $x_{n+1} = F(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given	There is a video tutorial here which students could watch independently or you could use it with a whole class: https://www.tes.com/teaching-resource/iteration-6201516 Iterative formulae are covered in the chapter already linked. It includes examples and activities for learners to try:

Learning objectives	Suggested teaching activities
rearrangement of an equation, to determine a root to a prescribed degree of accuracy (knowledge of the condition for convergence is not included, but candidates should understand that an iteration may fail to converge).	http://www.cimt.plymouth.ac.uk/projects/mepres/alevel/pure_ch19.pdf It is a good idea for learners to make full use of their calculator for the iteration process. Using the ANS (answer) key will save them time in finding a root of an equation. Here is an example and the method used to find a root. e.g. Using the iterative formula $x_{n+1} = 3 - \frac{1}{x_n}$ with $x_0 = 3$, show successive iterations to 5 decimal places and a final answer to 3 decimal places. • Start by entering the value of x_0 into the calculator: press 3 then '=' (or 'enter', depending on the calculator), so 3 appears as an answer. • Key in the right hand side of the iterative formula, replacing x_n with ANS (or the key that displays a previous answer) i.e. $3-(1 \div \mathrm{ANS})$. The calculator will display 2.666666667. Write this down to 5 decimal places. • Keep pressing the '=' key and successive iterations will appear. Write down as many as the question requires, all correct to 5 decimal places. 2.62500 2.61905 2.61818 2.61806 2.61804 2.61803 • You have now done enough iterations to show that an answer of 2.618 is correct to 3 decimal places. Learners will need practice at entering the correct formula into their calculator, using brackets where necessary. **Past papers: (I)(F)** Many of these questions involve other parts of the syllabus as well as this section. June 2014 paper 21, question 4 June 2014 paper 22, question 7 November 2014 paper 21, question 6 November 2014 paper 22, question 6 June 2013 paper 21, question 6 June 2013 paper 22, question 6

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Cambridge International Examinations
1 Hills Road, Cambridge, CB1 2EU, United Kingdom
tel: +44 1223 553554   fax: +44 1223 553558
email: info@cie.org.uk   www.cie.org.uk

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CAMBRIDGE ASSESSMENT