

Scheme of Work

Cambridge International AS & A Level Mathematics

9709/06 Probability & Statistics 1 (S1) For examination from 2017

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Introduction

This scheme of work provides ideas about how to construct and deliver a course. It has been broken down into different units of the three subject areas of Pure Mathematics (units P1, P2 and P3), Mechanics (units M1 and M2) and Probability & Statistics (units S1 and S2). For each unit there are suggested teaching activities and learning resources to use in the classroom for all of the syllabus learning objectives.

This scheme of work, like any other, is meant to be a guideline, offering advice, tips and ideas. It can never be complete but hopefully provides teachers with a basis to plan their lessons. It covers the minimum required for the Cambridge International AS & A Level course but also adds enhancement and development ideas. It does not take into account that different schools take different amounts of time to cover the Cambridge International AS & A Level course.

The mathematical content of Probability & Statistics 1 in the syllabus is detailed in the tables below. The order in which topics are listed is not intended to imply anything about the order in which they might be taught.

Recommended prior knowledge

Knowledge of the content of Cambridge O Level / Cambridge IGCSE® Mathematics is assumed.

Candidates will be expected to be familiar with scientific notation for the expression of compound units, e.g. 5 m s^{-1} for 5 metres per second.

As well as demonstrating skill in the appropriate techniques, candidates will be expected to apply their knowledge in the solution of problems. Individual questions set may involve ideas and methods from more than one section of the relevant content list.

Outline

Suggestions for independent study (**I**) and formative assessment (**F**) are indicated, where appropriate, within this scheme of work. The activities in the scheme of work are only suggestions and there are many other useful activities to be found in the materials referred to in the learning resource list.

Opportunities for differentiation are indicated as **basic/consolidation** and **challenging/extension**. There is the potential for differentiation by resource, length, grouping, expected level of outcome, and degree of support by the teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgment of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

Teacher support

Teacher Support (<http://teachers.cie.org.uk>) is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online.

This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on Teacher Support at <http://teachers.cie.org.uk>. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org.

Resources

The up-to-date resource list for this syllabus, including textbooks endorsed by Cambridge, is listed at www.cie.org.uk

Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. As such, all textbooks endorsed by Cambridge for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective.

Websites and videos

This scheme of work includes website links providing direct access to internet resources. Cambridge International Examinations is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services).

The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

1. Representation of data

Learning objectives	Suggested teaching activities
<p>Select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations may have.</p>	<p>When working with learners on questions that require them to draw statistical diagrams, it would be good practice for you to discuss with them why that particular diagram was chosen. For example, in these past papers:</p> <p>9709/63 June 14 question 1: why were back-to-back stem and leaf diagrams chosen? 9709/62 June 14 question 6: why was a histogram chosen?</p> <p>As an extension activity, learners could read the following interesting article from the University of Leicester. It covers suitable ways to present raw data, particularly focusing on how to present data when writing reports. http://www2.le.ac.uk/offices/ld/resources/numerical-data/numerical-data</p>
<p>Construct and interpret stem-and-leaf diagrams, box-and-whisker plots, histograms and cumulative frequency graphs.</p>	<p>You could show learners the following video links in class, pausing the video for learners to complete each task, or they could use the videos as an independent revision or consolidation resource.</p> <p>Constructing and interpreting stem and leaf diagrams from ExamSolutions. https://www.youtube.com/watch?v=RPEJl_Jkp4</p> <p>Introduction to histograms from ExamSolutions https://www.youtube.com/watch?v=CD4iaqtZTcA</p> <p>Box and Whisker plots explained (Marty Brandl) you-tube clip https://www.youtube.com/watch?v=635ErzR9Xzc</p> <p>Cumulative frequency – plotting https://www.youtube.com/watch?v=X9ajjBamRPc</p> <p>Past papers: (I)(F) Reading a histogram. 9709/62 November 2013 question 4 Constructing a histogram. 9709/62 June 2014 question 6 (ii) Constructing a stem-and-leaf diagram. 9709/61 November 2012 question 4 Cumulative frequency/box and whisker. 9709/63 November 2011 question 5</p>

Learning objectives	Suggested teaching activities
Understand and use different measures of central tendency (mean, median, mode) and variation (range, interquartile range, standard deviation), e.g. in comparing and contrasting.	<p>You could show learners the following video links in class, pausing the video for learners to complete each task, or they could use the videos as an independent revision or consolidation resource.</p> <p>A general introduction to measures of central tendency: https://www.khanacademy.org/math/probability/descriptive-statistics/central_tendency/v/statistics-intro-mean-median-and-mode</p> <p>Comparing different measures of central tendency with a discussion of which may be the best to use in certain situations: https://www.khanacademy.org/math/probability/descriptive-statistics/central_tendency/v/comparing-distribution-means https://www.khanacademy.org/math/probability/descriptive-statistics/central_tendency/v/exploring-mean-and-median-module</p> <p>General introduction to measures of dispersion: https://www.youtube.com/watch?v=E4HAYd0QnRc</p>
Use a cumulative frequency graph to estimate the median value, the quartiles and the interquartile range of a set of data.	<p>You could make a card matching activity for this topic involving cards with cumulative frequency diagrams on them for learners to match to the corresponding box and whisker plots. (F)</p> <p>Past paper question on cumulative frequency and box and whisker plots: 9709/63 November 2011 question 5</p>
Calculate the mean and standard deviation of a set of data (including grouped data) either from the data itself or from given totals such as Σx and Σx^2 , or $\Sigma(x - a)$ and $\Sigma(x - a)^2$.	<p>Textbooks will have many examples of questions that learners can use for calculation practice.</p> <p>Past papers: (I)(F) Calculating mean and standard deviation: 9709/62 November 2013 question 4 9709/62 June 2014 question 6(iii)</p>

2. Permutations and combinations

Learning objectives	Suggested teaching activities
<p>Understand the terms permutation and combination, and solve simple problems involving selections.</p>	<p>To help learners understand the terms permutation and combination, you could use these two video clips as a whole class activity, pausing the video at particular points to ask direct questions. (F) Alternatively, learners could watch the videos individually for revision or consolidation.</p> <p>Permutations, including notations used and the formula, are explained in the Khan Academy you-tube clip https://www.youtube.com/watch?v=XqQTXW7XfYA</p> <p>This video explains combinations and includes an explanation of the difference between permutations and combinations https://www.youtube.com/watch?v=bCxMhncR7PU</p>
<p>Solve problems about arrangements of objects in a line, including those involving:</p> <ul style="list-style-type: none"> -repetition (e.g. the number of ways of arranging the letters of the word 'NEEDLESS') -restriction (e.g. the number of ways several people can stand in a line if 2 particular people must – or must not – stand next to each other). 	<p>Past papers: (I)(F)</p> <p>9709/62 November 2013 question 6 9709/61 November 2012 question 7 9709/63 November 2011 question 4</p> <p>Questions and worked examples are on the Maths is Fun website: http://www.mathsisfun.com/combinatorics/combinations-permutations.html</p>

3. Probability

Learning objectives	Suggested teaching activities
<p>Evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events (e.g. for the total score when two fair dice are thrown), or by calculation using permutations or combinations.</p>	<p>You can perform experiments in class enabling learners to work out experimental probabilities and compare them with theoretical probabilities. (e.g. roll a die 120 times and see how many times it lands on each face. Repeat the experiment by rolling the die 600 times. Repeat with a 'biased' die; can learners work out how it is biased?) Probability 'kits' are widely available from educational suppliers, and often have biased dice and sample bottles in them that can make experiments quicker to perform.</p> <p>You could ask simple probability questions as a starter activity, with learners writing their answers on mini-whiteboards. (F)</p> <p>Some examples of practical activities and examples of evaluating probabilities in simple cases are at http://www.cimt.plymouth.ac.uk/projects/mepres/alevel/stats_ch1.pdf</p> <p>An extension activity (balls in a box) involving probabilities and tree diagrams is at http://www.s253053503.websitehome.co.uk/msv/msv-23.html</p> <p>You could use the following video links as interactive resources in class and you could pause the videos for direct questioning and for learners to complete tasks independently. Alternatively, learners could use the video independently as a revision or consolidation resource.</p> <p>Basic probability, equally probable events, and experimental probability are explained in the Khan Academy you-tube clip https://www.youtube.com/watch?v=uzkc-qNV0Ok</p> <p>You can find examples of calculating probabilities using combinations on the Khan Academy you-tube clip https://www.youtube.com/watch?v=Xqfcy1rqMbl</p> <p>There is a useful collection of Tarsia jigsaws available for free download from http://www.mrbartonmaths.com/jigsaw.htm. Go to Data and Statistical Topics then 'Probability Trees' to find one that will help learners to practise calculating probabilities from tree diagrams. (You may need to download the free Tarsia software first).</p> <p>An interactive (consolidation) resource on completing a probability tree diagram is here http://www.douis.net/liws/probability_tree.html</p>

Learning objectives	Suggested teaching activities
Use addition and multiplication of probabilities, as appropriate, in simple cases.	<p>You can find an explanation of the addition rule for probability on the Khan Academy you-tube clip https://www.youtube.com/watch?v=QE2uR6Z-NcU</p> <p>As a lesson starter activity, you could ask learners simple questions involving addition and multiplication of probabilities and they could answer on mini-whiteboards. (F)</p>
Understand the meaning of exclusive and independent events, and calculate and use conditional probabilities in simple cases, e.g. situations that can be represented by means of a tree diagram.	<p>You could show learners the following video clips as a whole class, or they could watch them independently. If you produce a set of questions covering key points from the videos, learners could answer them either while watching or afterwards. (F)</p> <p>This Khan Academy you-tube clip explains how to use tree diagrams to work out probabilities https://www.youtube.com/watch?v=6E_NVnboMB8</p> <p>The formula for conditional probabilities is explained, using both Venn diagrams and tree diagrams, on this you-tube clip https://www.youtube.com/watch?v=h05VK1XjVEY</p> <p>This is an interesting problem involving independent events and real data. You may wish to use it as an extension activity: http://s3.amazonaws.com/illustrativemathematics/illustration_pdfs/000/000/950/original/illustrative_mathematics_950.pdf?1390751089</p> <p>You could ask learners simple probability questions involving conditional probabilities as a lesson starter activity, with learners writing their answers on mini-whiteboards. (F)</p> <p>Past papers: (I)(F) 9709/62 November 2013 question 2 9709/62 November 2013 question 7(iii)(iv)</p>

4. Discrete random variables

Learning objectives	Suggested teaching activities
Construct a probability distribution table relating to a given situation involving a discrete random variable X , and calculate $E(X)$ and $\text{Var}(X)$.	<p>You could make a matching card activity consisting of a set of cards showing probability distribution tables for learners to match with the corresponding $E(X)$ and $\text{Var}(X)$. (F)</p> <p>Past papers: (I)(F) 9709/62 November 2013 question 7 (ii) 9709/61 November 2012 question 1 9709/62 June 2014 Qu 4</p>
Use formulae for probabilities for the binomial distribution, and recognise practical situations where the binomial distribution is a suitable model (the notation $B(n, p)$ is included).	<p>A Binostat is very useful for calculating binomial probabilities and deriving the formula. Balls are dropped into the binostat and learners can calculate the probability of a ball getting into each slot. You could show the video clip below as a demonstration of this: https://www.youtube.com/watch?v=ZQZy5834I2s</p> <p>You could use the following video links as interactive resources in class and you could pause the videos for direct questioning and for learners to complete tasks independently. Alternatively, learners could use the video independently as a revision or consolidation resource.</p> <p>You will find an explanation of the properties of a Binomial Distribution and the notation used in the ExamSolutions you-tube clip https://www.youtube.com/watch?v=NaDZ0zVTyXQ The formula is explained from a tree diagram on the clip https://www.youtube.com/watch?v=-U2cR-ErRVc</p> <p>Past papers: (I)(F) 9709/62 June 2014 question 1</p>
Use formulae for the expectation and variance of the binomial distribution.	<p>You could use the following video links as interactive resources in class, pausing the video for direct questioning and for learners to complete tasks independently. Alternatively, learners could use the video independently as a revision or consolidation resource.</p> <p>The formulae are given, and explained, and examples using them are in the ExamSolutions you-tube clip https://www.youtube.com/watch?v=zEyLaS2t8FI</p> <p>As an extension (challenging) activity, learners could consider the proof of the formulae. See the link here: http://www.s253053503.websitehome.co.uk/msv/msv-40.html</p>

5. The normal distribution

Learning objectives	Suggested teaching activities
Understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables.	<p>You could use the following video links as interactive resources in class, pausing the videos for direct questioning and for learners to complete tasks independently, or you could make a set of key questions to accompany the videos. (F) Alternatively, learners could use the videos independently as a revision or consolidation resource.</p> <p>The 'Standard Normal Distribution' is explained in the 'poysermath' you-tube clip https://www.youtube.com/watch?v=c11d3vVM5v8 and in the 'how2ststs' you-tube clip https://www.youtube.com/watch?v=xgQhefFOXrM</p> <p>This video explains how to find probabilities using tables https://www.youtube.com/watch?v=uxwkx4s7U18</p> <p>(Note that statistical tables can vary. Learners should be familiar with the tables given in the Cambridge International Examinations MF9 List of formulae and tables of the normal distribution.)</p> <p>There are some useful 'Tarsia' jigsaws to help learners to practise various calculations connected with the normal distribution. You can download them free from http://www.mrbartonmaths.com/jigsaw.htm if you click on Tarsia Applied Topics then Statistics: normal distribution. (You may need to download the Tarsia software from http://www.mmlsoft.com/index.php/products/tarsia)</p> <p>Past papers: (I)(F) 9709/62 November 2013 question 1</p>
Solve problems concerning a variable X , where $X \sim N(\mu, \sigma^2)$ including: -finding the value of $P(X > x)$, or a related probability, given the values of x, μ, σ -finding a relationship between x, μ , and σ given the value of $P(X > x)$ or a related probability.	<p>This ExamSolution you-tube clip presents a method for solving a problem involving finding the mean and standard deviation https://www.youtube.com/watch?v=CsuNZIQ-fsU</p> <p>Some examples, with support and hints are available here http://onlinestatbook.com/2/normal_distribution/ch6_exercises.html</p> <p>Past papers: (I)(F) 9709/61 November 2012 question 3</p>

Recall conditions under which the normal distribution can be used as an approximation to the binomial distribution (n large enough to ensure that $np > 5$ and $nq > 5$), and use this approximation, with a continuity correction, in solving problems.

You could use the following video links as interactive resources in class, pausing the videos for direct questioning and for learners to complete tasks independently, or you could make a set of key questions to accompany the videos. **(F)** Alternatively, learners could use the videos independently as a revision or consolidation resource.

An introduction to the Normal approximation to the Binomial from 'onlinestatbook'

http://onlinestatbook.com/2/normal_distribution/normal_approxM.html

Conditions for this approximation and the use of a continuity correction are discussed in the ExamSolutions you-tube clip <https://www.youtube.com/watch?v=SmjepW2Mb28>

© Cambridge International Examinations 2016
Version 1.1
Updated: 01.03.16

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