Scheme of Work

Cambridge International AS & A Level Mathematics

9709/07 Probability and Statistics 2 (S2)

For examination from 2017

Cambridge Advanced

CAMBRIDGE International Examinations

Contents

Introduction

This scheme of work provides ideas about how to construct and deliver a course. It has been broken down into different units of the three subject areas of Pure Mathematics (units P1, P2 and P3), Mechanics (units M1 and M2) and Probability & Statistics (units S1 and S2). For each unit there are suggested teaching activities and learning resources to use in the classroom for all of the syllabus learning objectives.

This scheme of work, like any other, is meant to be a guideline, offering advice, tips and ideas. It can never be complete but hopefully provides teachers with a basis to plan their lessons. It covers the minimum required for the Cambridge International AS & A Level course but also adds enhancement and development ideas. It does not take into account that different schools take different amounts of time to cover the Cambridge International AS & A Level course.

The mathematical content of Probability & Statistics 1 in the syllabus is detailed in the tables below. The order in which topics are listed is not intended to imply anything about the order in which they might be taught.

Recommended prior knowledge

Knowledge of the content of Cambridge O Level / Cambridge IGCSE® Mathematics is assumed.

Candidates will be expected to be familiar with scientific notation for the expression of compound units, e.g. 5 m s^{-1} for 5 metres per second.

As well as demonstrating skill in the appropriate techniques, candidates will be expected to apply their knowledge in the solution of problems. Individual questions set may involve ideas and methods from more than one section of the relevant content list.

Outline

Suggestions for independent study **(I)** and formative assessment **(F)** are indicated, where appropriate, within this scheme of work. The activities in the scheme of work are only suggestions and there are many other useful activities to be found in the materials referred to in the learning resource list.

Opportunities for differentiation are indicated as **basic/consolidation** and **challenging/extension**. There is the potential for differentiation by resource, length, grouping, expected level of outcome, and degree of support by the teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgment of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

Teacher support

Teacher Support (http://teachers.cie.org.uk) is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online.

This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on Teacher Support at http://teachers.cie.org.uk. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org.

Resources

The up-to-date resource list for this syllabus, including textbooks endorsed by Cambridge, is listed at www.cie.org.uk

Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. As such, all textbooks endorsed by Cambridge for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective.

Websites and videos

This scheme of work includes website links providing direct access to internet resources. Cambridge International Examinations is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services).

The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

1. The Poisson distribution

Learning objectives	Suggested teaching activities
Calculate probabilities for the distribution Po(n).	You could use a dominoes activity using Tarsia software. For a particular Poisson distribution, you could give learners cards to match, for example, P(X=6) to its answer **(F)**. You can download Tarsia software free of charge to create this type of activity http://www.mmlsoft.com/index.php/products/tarsia You may find the following video link useful in class, pausing the video for learners to complete each task. Alternatively, learners could use the videos as an independent revision or consolidation resource. This 'ExamSolutions' you-tube clip gives an explanation of the Poisson distribution and use of the formula https://www.youtube.com/watch?v=2zK3KpV3bx4
Use the fact that if $X \sim$ Po(n) then the mean and variance of X are each equal to n.	**Past papers: (I)(F)** 9709/72 November 2011 question 1(ii)
Understand the relevance of the Poisson distribution to the distribution of random events, and use the Poisson distribution as a model.	**Past papers: (I)(F)** 9709/72 November 2011 question 6(i)
Use the Poisson distribution as an approximation to the binomial distribution where appropriate ($n > 50$ and $np < 5$, approximately).	As a lesson starter activity, you could ask learners to answer on mini-whiteboards a few simple questions evaluating parameters and considering conditions for possible Poisson distributions. **(F)** You could show learners the following video links in class, pausing the video for learners to carry out some of the calculations themselves, or they could use the videos as an independent revision or consolidation resource. An example of the approximation is shown on the you-tube clip from 'ExamSolutions' https://www.youtube.com/watch?v=nZgnov87jVM
Use the normal distribution, with continuity correction, as an approximation to the Poisson distribution where appropriate ($n > 15$, approximately).	As a lesson starter activity, you could ask learners to answer on mini-whiteboards simple questions evaluating parameters and considering conditions. **(F)** You could show learners the following video link in class, pausing the video for learners to carry out some of the

Learning objectives	Suggested teaching activities
	calculations themselves, or they could use the videos as an independent revision or consolidation resource. You could prepare in advance a set of key questions to accompany the video clip **(F)** The Normal approximation to the Poisson distribution, with conditions and explanation of continuity corrections, is illustrated on the you-tube clip from 'ExamSolutions https://www.youtube.com/watch?v=7RuoHKhzs2I (Note that learners should be familiar with the conditions needed to use this approximation. These are only approximate values: the video clip uses $n > 20$, but $n > 15$ is sufficient for this syllabus.) **Past papers: (I)(F)** 9709/72 November 2011 question 6 9709/72 June 2014 question 3

2. Linear combinations of random variables

Learning objectives	Suggested teaching activities
Use, in the course of solving problems, the results that: - E($aX + b$) = aE(X) + b and Var($aX + b$) = a^2Var(X) - E($aX + bY$) = aE(X) + bE(Y) - Var($aX + bY$) = a^2Var(X) + b^2Var(Y) for independent X and Y - if X has a normal distribution then so does $aX + b$ - if X and Y have independent normal distributions then $aX + bY$ has a normal distribution -if X and Y have independent Poisson distributions then $X + Y$ has a Poisson distribution.	This Khan Academy you-tube video covers linear combinations https://www.youtube.com/watch?v=rLdoKZ7w0xI You could show this as a whole class activity, pausing the video for learners to complete tasks independently. Alternatively, learners could watch the video individually as a revision or consolidation resource. To help learners to understand the difference between the distributions 2X and $X_1 + X_2$, (i.e. when to multiply and when to add), you could carry out an experiment. For example, consider the question: how does 'rolling a die and doubling' differ from 'rolling two dice and adding'? See this link: http://www.s253053503.websitehome.co.uk/msv/msv-19.html Learners can obtain further practice using 'Tarsia' dominoes Linear Combinations 3**,** available for free download from http://www.mrbartonmaths.com/jigsaw.htm if you click on Tarsia Applied Topics then Statistics: linear combinations. To demonstrate adding two independent Poisson variables, you could show this presentation http://www.s253053503.websitehome.co.uk/msv/msv-38.html **Past papers: (I)(F)** 9709/73 November 2011 question 1 9709/72 November 2011 question 1 9709/72 November 2013 question 7

3. Continuous random variables

Learning objectives	Suggested teaching activities
Understand the concept of a continuous random variable, and recall and use properties of a probability density function (restricted to functions defined over a single interval).	**Past papers: (I)(F)** 9709/72 November 2013 question 5
Use a probability density function to solve problems involving probabilities, and to calculate the mean and variance of a distribution (explicit knowledge of the cumulative distribution function is not included, but location of the median, for example, in simple cases by direct consideration of an area may be required).	A card matching activity could be useful here: you could make cards for learners to match a given probability density function to the corresponding mean, variance etc. **(F)** You can download Tarsia software free of charge and use it to create a matching activity http://www.mmlsoft.com/index.php/products/tarsia **Past papers: (I)(F)** 9709/72 November 2011 question 7 9709/72 June 2014 question 6

4. Sampling and estimation

Learning objectives	Suggested teaching activities
Understand the distinction between a sample and a population, and appreciate the necessity for randomness in choosing samples.	**Past papers: (I)(F)** 9709/73 November 2011 question 3(i)(ii)
Explain in simple terms why a given sampling method may be unsatisfactory (knowledge of particular sampling methods, such as quota or stratified sampling, is not required, but candidates should have an elementary understanding of the use of random numbers in producing random samples).	When working on questions that require samples to be picked, you could discuss with learners why particular methods were chosen. Alternatively, if the question does not suggest a particular method, you could ask them what would be a suitable method. As a class activity you could generate a random sample experimentally from a parent population using a suitable sampling method. **Past papers: (I)(F)** 9709/72 June 2014 question 7(i) 9709/73 November 2011 question 3(i)
Recognise that a sample mean can be regarded as a random variable, and use the facts that $E(\bar{X})=\mu$ and that $Var(\bar{X}) = \sigma^2/n$.	Learners will need to practise calculating parameters for the distribution of the sample means. An engaging activity you can use is in class is the 'Tarsia' jigsaw 'Sample means and the central limit theorem'. Go to http://www.mrbartonmaths.com/jigsaw.htm, click on Tarsia Applied Topics then Central Limit Theorem to download the activity free of charge.
Use the fact that $\bar{X}$ has a normal distribution if X has a normal distribution.	You could demonstrate this experimentally using an underlying normal distribution: take samples of the same size from the given normally distributed population and plot the distribution of these sample means. Then take samples of a larger size from the population and carry out the experiment again. The following video would be an interesting extension activity for some learners: http://onlinestatbook.com/2/normal_distribution/history_normalM.html
Use the Central Limit Theorem where appropriate.	You can carry out the same experiment as above with a non-normally distributed parent population. This illustrates that the sample means follow a normal distribution regardless of the underlying population distribution as long as the sample size is large enough. Learners need to be aware of when it is appropriate to use the Central Limit Theorem. Note that n>30 is considered 'large' for the purpose of this syllabus.

Learning objectives	Suggested teaching activities
	The experiment can also be run as a simulation using onlinestatbook.com http://onlinestatbook.com/2/sampling_distributions/SampDist_v1.html#video or the distribution maker here: http://nrich.maths.org/content/id/5932/stage_distributions.swf You could show learners the following video links in class, pausing each video for learners to carry out some of the calculations themselves, or they could use the videos as an independent revision or consolidation resource. You could prepare in advance a set of key questions to accompany the video clips. **(F)** For an introduction to sampling distributions http://onlinestatbook.com/2/sampling_distributions/intro_samp_distM.html The Central Limit Theorem is illustrated on the Khan Academy you-tube clips https://www.youtube.com/watch?v=JNm3M9cqWyc and https://www.youtube.com/watch?v=FXZ2O1Lv-KE. This video clip discusses the variance https://www.youtube.com/watch?v=NYd6wzYkQIM. Learners can practise calculating parameters for the distribution of the sample means using the 'Tarsia' jigsaw 'Sample means and central limit theorem'. Go to http://www.mrbartonmaths.com/jigsaw.htm, click on Tarsia Applied Topics then Central Limit Theorem to download the activity free of charge. **Past papers: (I)(F)** 9709/72 June 2014 question 7(iv)
Calculate unbiased estimates of the population mean and variance from a sample, using either raw or summarised data (only a simple understanding of the term 'unbiased' is required).	You could use card matching activity could be used here: give learners cards that each show a different set of raw data or summarised data to match with corresponding cards showing biased and unbiased estimates. **(F)** You can download Tarsia software free of charge and use it to create a matching activity: http://www.mmlsoft.com/index.php/products/tarsia **Past papers: (I)(F)** 9709/72 November 2011 question 4(i) **Past papers: (I)(F)** 9709/72 November 2013 question 3(i)

Learning objectives	Suggested teaching activities
Determine a confidence interval for a population mean in cases where the population is normally distributed with known variance or where a large sample is used.	This 'jbstatistics' you-tube clip discusses constructing and interpreting a confidence interval for the population mean https://www.youtube.com/watch?v=KG921rfbTDw You could show learners the video link in class, pausing the video for learners to carry out some of the calculations themselves, or they could use it as an independent revision or **consolidation** resource. Learners can practise finding the *z* value when constructing a confidence interval using the 'Tarsia' jigsaw 'Confidence Intervals'. Go to http://www.mrbartonmaths.com/jigsaw.htm click on Tarsia Applied Topics then Confidence Intervals to download the activity free of charge. **Past papers: (I)(F)** 9709/72 November 2011 question 4(ii)(iii)
Determine, from a large sample, an approximate confidence interval for a population proportion.	You could create a 'spot the error' activity sheet. Give some incorrect solutions to learners and ask them to 'mark' the work then discuss the errors. (You can incorporate errors deliberately into the solutions, based on common misconceptions). You could show learners the video link below in class, pausing the video for learners to complete tasks independently or to discuss key points. Alternatively, they could use it as an independent revision or consolidation resource. This StatisticsLectures you-tube clip shows how to construct a confidence interval for a population proportion: https://www.youtube.com/watch?v=3ReWri_jh3M **Past papers: (I)(F)** 9709/72 June 2014 question 5

5. Hypothesis tests

Learning objectives	Suggested teaching activities
Understand the nature of a hypothesis test, the difference between one-tail and two-tail tests, and the terms null hypothesis, alternative hypothesis, significance level, rejection region (or critical region), acceptance region and test statistic.	You could show learners the video links below in class, pausing the videos for questioning or for learners to complete tasks independently. Alternatively, they could use it as an independent revision or consolidation resource. For an explanation of null and alternative hypotheses and one and two-tail test https://www.youtube.com/watch?v=W6Wn_J5qWVw For an explanation of type I and type II errors see the 'jbstatistics' you-tube clip https://www.youtube.com/watch?v=7mE-K_w1v90 (Note only the first part of this clip is relevant – the power of the test is not required for this syllabus.) You could also use a card matching activity could be used here: give learners cards that each show a given scenario and ask them to match them with corresponding cards showing null and alternative hypotheses. **(F)**
Formulate hypotheses and carry out a hypothesis test in the context of a single observation from a population which has a binomial or Poisson distribution, using either direct evaluation of probabilities or a normal approximation, as appropriate.	You could show learners the following video links in class, pausing the videos for learners to carry out some of the calculations themselves, or they could use the videos as an independent revision or consolidation resource. You could prepare in advance a set of key questions to accompany the video clips **(F)** Hypothesis testing for a Poisson Distribution is explained in the ExamSolution you-tube clip https://www.youtube.com/watch?v=48gjjjaCVYw Hypothesis testing for a Binomial Distribution is explained in the ExamSolution you-tube clip https://www.youtube.com/watch?v=61Wi04SqF34 **Past papers: (I)(F)** 9709/72 November 2011 question 2 (Binomial population approximated to Normal)
Understand the terms Type I error and Type II error in relation to hypothesis tests.	An interesting article for learners to read, "Which type of error would you prefer?", might help them to distinguish and remember the two types of error https://learnandteachstatistics.wordpress.com/2012/10/29/which-type-of-error-do-you-prefer/ **Past papers: (I)(F)** 9709/72 November 2013 question 6

Learning objectives	Suggested teaching activities
Calculate the probabilities of making Type I and Type II errors in specific situations involving tests based on a normal distribution or direct evaluation of binomial or Poisson probabilities.	**Past papers: (I)(F)** 9709/72 November 2011 question 5

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Cambridge International Examinations
1 Hills Road, Cambridge, CB1 2EU, United Kingdom
tel: +44 1223 553554 fax: +44 1223 553558
email: info@cie.org.uk www.cie.org.uk