Scheme of Work

Cambridge International AS & A Level

Cambridge International AS & A Level Mathematics

9709/01 Pure Mathematics 1 (P1)

For examination from 2017

Cambridge Advanced

CAMBRIDGE International Examinations

Contents

Introduction

This scheme of work provides ideas about how to construct and deliver a course. It has been broken down into different units of the three subject areas of Pure Mathematics (units P1, P2 and P3), Mechanics (units M1 and M2) and Probability & Statistics (units S1 and S2). For each unit there are suggested teaching activities and learning resources to use in the classroom for all of the syllabus learning objectives.

This scheme of work, like any other, is meant to be a guideline, offering advice, tips and ideas. It can never be complete but hopefully provides teachers with a basis to plan their lessons. It covers the minimum required for the Cambridge International AS & A Level course but also adds enhancement and development ideas. It does not take into account that different schools take different amounts of time to cover the Cambridge International AS & A Level course.

The mathematical content of Pure Mathematics 1 in the syllabus is detailed in the tables below. The order in which topics are listed is not intended to imply anything about the order in which they might be taught.

Recommended prior knowledge

Knowledge of the content of Cambridge O Level / Cambridge IGCSE® Mathematics is assumed.

Candidates will be expected to be familiar with scientific notation for the expression of compound units, e.g. 5 m s^{-1} for 5 metres per second.

As well as demonstrating skill in the appropriate techniques, candidates will be expected to apply their knowledge in the solution of problems. Individual questions set may involve ideas and methods from more than one section of the relevant content list.

Outline

Suggestions for independent study **(I)** and formative assessment **(F)** are indicated, where appropriate, within this scheme of work. The activities in the scheme of work are only suggestions and there are many other useful activities to be found in the materials referred to in the learning resource list.

Opportunities for differentiation are indicated as **basic/consolidation** and **challenging/extension**. There is the potential for differentiation by resource, length, grouping, expected level of outcome, and degree of support by the teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgment of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

Teacher support

Teacher Support (http://teachers.cie.org.uk) is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online.

This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on Teacher Support at http://teachers.cie.org.uk. If you are unable to use Microsoft Word you can download Open Office free of charge from www.openoffice.org.

Resources

The up-to-date resource list for this syllabus, including textbooks endorsed by Cambridge, is listed at www.cie.org.uk

Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. As such, all textbooks endorsed by Cambridge for this syllabus are the ideal resource to be used alongside this scheme of work as they cover each learning objective.

Websites and videos

This scheme of work includes website links providing direct access to internet resources. Cambridge International Examinations is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services).

The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

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1. Quadratics

Learning objectives	Suggested teaching activities
Carry out the process of completing the square for a quadratic polynomial $ax^2 + bx + c$ and use this form, e.g. to locate the vertex of the graph of $y = ax^2 + bx + c$ or to sketch the graph.	Most of this section on quadratics should be already familiar to learners, so activities that review work done at IGCSE Maths or equivalent should be considered. A good starting point would be a quick summary on solving quadratic equations by factorisation and using the quadratic formula. Learners should have completed the square for IGCSE so you could review this using http://www.mrbartonmaths.com/c1.htm - look for 'Quadratic Equations and Graphs' and then 'quadratic graphs' (Excel). You could follow this with an activity which will reinforce the basic concepts. https://www.tes.com/teaching-resources has many useful resources for practising the basic method. Search for 'Completing the square' and, for example, find the yellow card uploaded by 'headofslytherin' then print the resource cards.
Find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant, e.g. to determine the number of real roots of the equation $ax^2 + bx + c = 0$.	Introduce the idea of the discriminant of a quadratic expression, $b^2 – 4ac$, and show that it comes from the quadratic formula for solving $ax^2 + bx + c = 0$. By displaying the quadratic formula on a board, you can encourage learners to identify the three different values that the discriminant can take and to relate those to the number of roots of a particular quadratic equation. http://www.purplemath.com/modules/quadform3.htm provides a demonstration of the three different situations.
Solve quadratic equations, and linear and quadratic inequalities, in one unknown.	The previous two sections will have covered solving quadratic equations. http://www.bbc.co.uk/bitesize/quiz/q99393657 provides a quick test on solving quadratic equations using different methods. For solving linear inequalities, a good resource is http://www.purplemath.com/modules/ineqsolv.htm which makes it clear that you can treat linear inequalities in a similar way to linear equations. It emphasises that multiplying or dividing throughout by a negative number reverses the inequality. (You can instead consider addition or subtraction of terms to move them to the other side of the inequality.) http://tube.geogebra.org/student/m92032 is a useful activity which learners can use to practise the basic solution of linear inequalities and check their answers. **(I)** When solving a quadratic inequality, it is important to obtain the critical values first and then to determine the correct

Learning objectives	Suggested teaching activities
	set of values for *x* that satisfies the inequality. Finding the critical values by solving the equation gives learners a chance to consolidate work done in the previous sections. You can encourage learners to sketch the graph of the quadratic function to help them solve the inequality. There is an interesting puzzle resource here https://www.tes.com/teaching-resource/tarsia-quadratic-inequalities-2-6110158 which you can download (log in free of charge) and https://www.tes.co.uk/teaching-resource/solving-quadratic-inequalities-worksheet-6026905 provides a useful worksheet. **(I)** **Past papers: (I)(F)** June 2011 paper 11 question 2 June 2014 paper 13 question 8
Solve by substitution a pair of simultaneous equations of which one is linear and one is quadratic.	You can start with fairly straightforward examples with an obvious substitution e.g. $2x^2 - 3x - y = 1$ and $y = 3x + 7$. http://www.bbc.co.uk/schools/gcsebitesize/maths/algebra/quadequationshirev4.shtml provides good worked examples using graphs. **Past papers: (I)(F)** November 2011 paper 11 question 9 November 2011 paper 12 question 4 In these questions, the technique is only needed as part of the question
Recognise and solve equations in *x* which are quadratic in some function of *x*, e.g. $x^4 - 5x^2 + 4 = 0$.	The equation given in this learning objective provides a good starting point for this section. Throughout the course, you could introduce quadratic equations of different types when you reach the appropriate part of the syllabus, e.g. $\mathrm{e}^{2x} - 7\mathrm{e}^x + 12 = 0$, $2\left(3^{2x+1}\right) - 5\left(3^x\right) + 1 = 0$, $3\tan^2 x - \tan x - 2 = 0$. **Past papers: (I)(F)** June 2011 paper 11 question 5 November 2011 paper 13 question 3

2. Functions

Learning objectives	Suggested teaching activities
Understand the terms function, domain, range, one-one function, inverse function and composition of functions.	Start by defining the terms function, domain and range. The following link gives an informal way of helping learners to understand the meaning of these terms. http://www.coolmath.com/algebra/15-functions/01-whats-a-function-domain-range-01 You will cover definitions of the terms one-one function, inverse function and composition of functions in later sections, together with the appropriate notation.
Identify the range of a given function in simple cases, and find the composition of two given functions.	Start with some simple functions with straightforward domains and ask learners to identify the ranges for each function e.g. $\mathrm{f}(x) = 5x + 1,\ x \in \mathbb{R}$, $\mathrm{f}(x) = 5x + 1,\ x \geq 2$. You can encourage learners to sketch graphs to help them. Move on to examples where learners could be required to choose a suitable domain and hence find the corresponding range. e.g. $y = \sqrt{x+3}$, $\mathrm{f}(x) = \dfrac{1}{\sqrt{2x-5}}$ Introduce the idea of composite functions by considering two separate functions, f and g, as rules for converting input into output, e.g. $\mathrm{f}(x) = 2x - 3,\ x \in \mathbb{R}$ and $\mathrm{g}(x) = 1 - 2x,\ x \in \mathbb{R}$. Ask learners to consider an input of $x = 2$, for example, and to apply the rule for f and then the rule for g to the result. Repeat the process but with the rule for g applied first, followed by the rule for f. Introduce the notation used and emphasise the importance of the order. Go to http://www.mathworksheetsgo.com/ then search for 'Composition of Functions' to find a link to an interactive lesson showing the importance of the order of carrying out functions. There is another link to a worksheet of suitable questions. **(I)(F)** As you work through the syllabus, you may wish to return to this topic when considering trigonometric functions, exponential functions and logarithmic functions.
Determine whether or not a given function is one-one, and find the inverse of a one-one function in simple cases.	Learners need to classify different types of functions. Start with some straight lines, quadratic curves and cubic curves. Go to http://www.mathworksheetsgo.com/ then search for 'One to one functions' to find a resource on how to identify one-one functions. There is also a worksheet called '1 to 1 functions' for practice. **(I)(F)** This will lead conveniently to introducing the inverse of a one-one function. Start with some simple one-one functions

Learning objectives	Suggested teaching activities
	and ask learners to state the inverse functions e.g. $y = x + 2$, $y = 3x$, $y = x - 7$ and $y = \frac{x}{7}$ (all with a domain $x \in \mathbb{R}$). Move on to some more complicated one-one functions and see if learners can deduce a quick way of obtaining the inverses. Introduce the idea of restricting the domain of a function so that a many-one function becomes a one-one function over a certain domain. Go to http://www.mathworksheetsgo.com/ then search for 'Inverse functions' to locate a resource on finding the inverse and a worksheet on inverse functions. **(I)(F)**
Illustrate in graphical terms the relation between a one-one function and its inverse.	Start with a straightforward function that learners could easily sketch e.g. $y = 2x + 1,\ x \in \mathbb{R}$. Ask them to work out the inverse of the function and also to sketch this. You can then ask them to consider the result of $\mathrm{f} \circ \mathrm{f}^{-1}$ and $\mathrm{f}^{-1} \circ \mathrm{f}$. This could be repeated with a more complicated function e.g. $\mathrm{f}(x) = \sqrt{x - 5},\ x \geq 5$. Ask learners to present their results and conclusions. This link provides a demonstration of the important points. It is followed by some questions http://www.mathsisfun.com/sets/function-inverse.html **(I)** **Past papers: (I)(F)** June 2014 paper 11, question 10 June 2014 paper 12, question 10 November 2014 paper 12, question 11 November 2014 paper 13, question 10(a) June 2013 paper 11, question 8 (also makes use of completing the square) June 2013 paper 13, question 10

3. Coordinate geometry

Learning objectives	Suggested teaching activities
Find the length, gradient and mid-point of a line segment, given the coordinates of the end-points.	This should be consolidation/revision of work done previously. For a quick review of this work, give learners the coordinates of the end points of different line segments and ask them to find the length, gradient and midpoint of each line segment. **Past papers: (I)(F)** June 2014 paper 11, question 7 November 2014 paper 11, question 4
Find the equation of a straight line given sufficient information (e.g. the coordinates of two points on it, or one point on it and its gradient).	Learners will have met the equation of a straight line before, so again this should be a revision exercise. Click on http://www.mathsisfun.com/index.htm and search for 'Equation of a straight line' to find a page of examples and graphs illustrating them. There is an interactive part which allows learners to explore different gradients and intercepts. This is followed by a selection of questions for consolidation.
Understand and use the relationships between the gradients of parallel and perpendicular lines	For more able learners, you could demonstrate the proof that the product of the gradients of perpendicular lines is -1. This interactive link provides basic practice in working with parallel and perpendicular lines. It could be useful as a revision exercise as answers are checked automatically. http://www.cimt.plymouth.ac.uk/projects/mepres/book9/bk9i5/bk9_5i4.html This link is similar but uses US terminology ('slope' instead of 'gradient'). http://uk.ixl.com/math/year-12/slopes-of-parallel-and-perpendicular-lines **Past papers: (I)(F)** June 2014 paper 12, question 1 November 2014 paper12, question 9 November 2014 paper 13, question 3 June 2013 paper 13, question 7
Interpret and use linear equations, particularly the forms $y = mx + c$ and $y - y_1 = m(x - x_1)$.	This section consolidates the work in the previous sections. Learners should be able to use both forms of the straight line equation. Here is a worksheet which covers all of the syllabus requirements for this topic (log in free of charge) https://www.tes.co.uk/teaching-resource/a-level-maths-c1-coordinate-geometry-worksheet-6135231 **(I)(F)**

Learning objectives	Suggested teaching activities
Understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations (including, in simple cases, the correspondence between a line being tangent to a curve and a repeated root of an equation).	Most learners will already be familiar with points of intersection of straight lines. As a revision exercise, you could use http://www.mathopenref.com/coordintersection.html which provides an interactive resource allowing you to change the equations of the straight line graphs. You could give learners an example to investigate e.g. the intersection of the line $y = 4x - 9$ and the curve $y = 4x\left(x-2\right)$. They could use this to explore the link between a line as tangent to a curve, the repeated root of an equation and the discriminant. Other examples could be considered, for example: Given that the line $y = mx - 3$ is a tangent to the curve $y = x^2 - 5x + 6$, find the value of *m*. **Past papers: (I)(F)** June 2013 paper 12, question 7

4. Circular measure

Learning objectives	Suggested teaching activities
Understand the definition of a radian, and use the relationship between radians and degrees.	Ask learners to calculate the angle, in degrees, subtended by an arc, of length 1 unit, of a circle which has radius 1 unit. This will illustrate the definition of a radian and give learners an idea of the approximate size of a radian in degrees. You can then present more formally the relationship between radians and degrees in terms of π and learners can practice converting degrees to radians using this link https://www.khanacademy.org/math/trigonometry/unit-circle-trig-func/radians_tutorial/e/degrees_to_radians
Use the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ in solving problems concerning the arc length and sector area of a circle.	To obtain the formula for length of arc you can consider a circle of radius r and an arc of length s that subtends an angle θ radians at the centre of the circle of circumference $2\pi r$. You could ask learners to find the length of the arc using the ratio of the angles, $\frac{\theta}{2\pi}$, and to derive the formula for the sector area of a circle using a similar method. Go to http://www.mathsisfun.com/index.htm and search for 'circle sector' to find Circle Sector and Segment, a tutorial and practice questions on arc lengths and areas of sector. There are various different geometrical approaches that may be used in questions. Learners need to practise as many questions as possible so that they become as familiar with radians as with degrees. You could also usefully remind them about the formula for the area of a triangle. **Past papers: (I)(F)** June 2014 paper 11, question 6 June 2014 paper 12, question 4 June 2014 paper 13, question 3 November 2014 paper 11, question 8 November 2014 paper 12, question 2 November 2014 paper 13, question 2

5. Trigonometry

Learning objectives	Suggested teaching activities
Sketch and use graphs of the sine, cosine and tangent functions (for angles of any size, and using either degrees or radians).	An interactive graph plotter is very helpful for showing different trigonometric curves. All of these are free resources: https://www.desmos.com/calculator (register to get free examples such as Trigonometry – unit circle), https://www.geogebra.org/ (free to download) http://rechneronline.de/function-graphs/, http://graph-plotter.cours-de-math.eu/ You can go to https://nrich.maths.org/ and search for ‘Sine problem’ to find an interesting pattern generated by transformations of sin *x*. The problem involves naming the functions used to generate the pattern and provides useful practice on visualising the graphs.
Use the exact values of the sine, cosine and tangent of 30°, 45°, 60°, and related angles, e.g. cos 150° = - ½√3.	Most scientific calculators will now give exact values of sin, cos and tan of the given angles. You can use an equilateral triangle of side 2 units to find the exact values of sin, cos and tan of 30° and 60°, and a right-angled isosceles triangle of sides 1, 1, √2 to find the exact values of sin, cos and tan of 45°. For the trigonometric ratios of angles in other quadrants you could consider projections on to the *x*-axis as shown here: http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-trigratiosanysize-2009-1.pdf Alternatively, you could deduce the results from the trigonometric graphs. It is a great advantage for learners to be able to understand and use the graphs, for example in solving equations. **Past papers: (I)(F)** June 2014 paper 12, question 3
Use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometric relations.	Learners should already be familiar with the notation used, but it is worth reinforcing the principal values for each trigonometric ratio. For example $-90^\circ \leqslant \theta \leqslant 90^\circ$ gives the principal value for the solution of $\sin\theta = k,\ -1 \leqslant k \leqslant 1$. **Past papers: (I)(F)** November 2014 paper 11, question 2

Learning objectives	Suggested teaching activities
Use the identities $\dfrac{\sin\theta}{\cos\theta} \equiv \tan\theta$ and $\sin^2\theta + \cos^2\theta \equiv 1$.	You could introduce the identities using a right-angled triangle. Alternatively you can download a worksheet and a matching exercise here (log in free of charge): https://www.tes.co.uk/teaching-resource/a-level-maths-trigonometry-identities-worksheets-6146808 For exam questions, see the next section.
Find all the solutions of simple trigonometrical equations lying in a specified interval (general forms of solution are not included).	To obtain all the solutions for a given equation, you can encourage learners to use the graphs of the trigonometric functions. The link below demonstrates this (log in for free download): https://www.tes.co.uk/teaching-resource/solving-trig-equations-6336755 Another approach is to make use of the unit circle. https://www.tes.co.uk/teaching-resource/cast-diagram-for-solving-trigonometric-equations-6332281 Both links provide examples and most A Level maths text books will have many examples. You can set learners a mixture of questions in radians and degrees and remind them to check their calculator is in the correct mode. Many exam questions involve a proof or 'show that' exercise, the result of which is then used in solving a trigonometric equation. Most of the questions listed below are of this type. **Past papers: (I)(F)** June 2014 paper 11, question 9 June 2014 paper 12, question 5 June 2014 paper 13, question 4 November 2014 paper 11, question 3 November 2014 paper 12, question 5 November 2014 paper 13, question 3 June 2013 paper 11, question 5 June 2013 paper 12, question 5 June 2013 paper 13, question 3

6. Vectors

Learning objectives	Suggested teaching activities
• use standard notations for vectors, i.e. $\begin{pmatrix} x \\ y \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j}$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $\overrightarrow{AB}$, $\mathbf{a}$.	You could start by asking learners to give examples of vector and scalar quantities, with explanations. Introduce the notation for the coordinate axes, first in two dimensions then extending to three dimensions, and then follow on with the notation for vectors. Encourage learners to use correct vector notation when working through problems. The link below leads to a quick practice exercise. http://www.bbc.co.uk/schools/gcsebitesize/maths/geometry/vectorshirev1.shtml Learners may find it easier to use vectors in column vector form rather than component form. The magnitude of a vector quantity could also be introduced at this point.
Carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms.	Many learners will already be familiar with adding and subtracting vectors so a revision activity may be a good starting point. This would also help those meeting vectors for the first time. Go to http://www.mathsisfun.com/ and search for 'Vectors' to find a useful set of diagrams explaining vector notation, addition and subtraction. You could encourage learners to study this independently and come to the lesson ready to start solving problems. **(I)** There are examples of geometrical problems involving addition and subtraction here: http://www.bbc.co.uk/schools/gcsebitesize/maths/geometry/vectorshirev2.shtml You could go to https://nrich.maths.org and search for 'Vector Walk' to find a problem involving addition and subtraction.
Use unit vectors, displacement vectors and position vectors.	Many learners have difficulties with the concept of a unit vector. Start by drawing some vectors on graph paper or a board, e.g. $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 5 \\ -12 \end{pmatrix}$. Show how the first vector can be 'split' into 5 separate vectors each of length one unit. Verify that the same principle applies to the second vector. Ask the learners to deduce how to find a unit vector. This will lead on to finding the magnitude of a vector which comes in the next section, so you will need to consider the order in which you teach topics. Extend this work to involve 3-dimensional vectors. Define a displacement vector as follows: The **displacement** of an object is defined as the **vector distance** from an initial point to a final point. It is important for learners to understand that this is different from the **distance travelled**.

Learning objectives	Suggested teaching activities
	You can also point out to learners that direction is crucial, e.g. displacement vector $\begin{pmatrix}2\\-5\end{pmatrix}$ is different from displacement vector $\begin{pmatrix}-2\\5\end{pmatrix}$, although the distance travelled is the same in both cases. Learners need to be aware of the unique nature of position vectors, e.g. a point *A* has position vector $\begin{pmatrix}2\\-1\\4\end{pmatrix}$ relative to an origin *O* so there is only one possible position for point *A*. For further practice using vectors, the link below (log in for free download) provides some good examples. Learners could work in groups on separate questions and present their answers to the class as a whole to check their method and use of notation. https://www.tes.co.uk/teaching-resource/vectors-worksheet-higher-gcse-6176054
Calculate the magnitude of a vector and the scalar product of two vectors.	The magnitude of a vector needs to be dealt with fairly early on in the topic. Many learners may have come across it already. Start with vectors such as $\begin{pmatrix}3\\4\end{pmatrix}$, $\begin{pmatrix}5\\-12\end{pmatrix}$, and ask learners to work out the magnitude, using a sketch if necessary. Go on to other vectors e.g. $\begin{pmatrix}4\\-5\end{pmatrix}$ and ensure that the learners give the magnitude in surd form, $\sqrt{41}$ in this case, rather than as a rounded decimal. Exact forms will often be required as answers to problems. You could introduce the scalar product of two vectors with a formal definition. Ask learners to work out the angle between two vectors in two dimensions, e.g. $\begin{pmatrix}5\\2\end{pmatrix}$ and $\begin{pmatrix}3\\12\end{pmatrix}$, using trigonometry and hence to work out the scalar product $\left\|\begin{pmatrix}5\\2\end{pmatrix}\right\|\left\|\begin{pmatrix}3\\12\end{pmatrix}\right\|\cos\theta$, where θ is the angle they have just found. Ask them if

Learning objectives	Suggested teaching activities
	they can deduce a quicker process of getting the scalar product. A formal proof (for two or three dimensions, or both) could follow after learners have worked out the scalar products of pairs of unit vectors such as $\mathbf{i}$ and $\mathbf{j}$, $\mathbf{i}$ and $\mathbf{k}$. The following link provides a good overview of the scalar product. Go to https://www.mathsisfun.com then search for 'Dot product'.
Use the scalar product to determine the angle between two directions and to solve problems concerning perpendicularity of vectors.	Show learners that the formula for the scalar product may be re-written as $\cos\theta = \frac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert.\lvert\mathbf{b}\rvert}$ and use it to find angles in triangles and parallelograms. Learners should be able to state the requirement for perpendicularity without much trouble. Most text books will have plenty of examples for practice and the following link provides a worksheet with solutions (log in for free download): https://www.tes.co.uk/teaching-resource/c4-maths-vectors-worksheet-6096103 **(I, F)** These exam paper questions provide a good cross-section of the type of questions that may be asked. Questions involve all of the techniques in the previous sections. **Past papers: (I)(F)** June 2014 paper 11, question 8 June 2014 paper 12, question7 June 2014 paper 113 question 7 November 2014 paper 11, question 6 November 2014 paper 12, question 7 November 2014 paper 13, question 7 June 2013 paper 11, question 6 June 2013 paper 12, question 6 June 2014 paper 13, question 8

7. Series

Learning objectives	Suggested teaching activities
Use the expansion of $(a + b)^n$, where n is a positive integer (knowledge of the greatest term and properties of the coefficients are not required, but the notations $\binom{n}{r}$ and $n!$ should be known).	You could start by asking learners to expand $(a+b)^2$, $(a+b)^3$ and $(a+b)^4$, setting out their work logically with powers of *a* decreasing and powers of *b* increasing, and to spot the pattern from Pascal's triangle. (Learners could do this in groups or individually.) They can then deduce the results for $(a+b)^5$ and $(a+b)^6$ without having to work through any expansions. Using the same pattern and Pascal's triangle coefficients, learners should be able to expand an expression such as $(2x-3y)^4$. A very common error is to write $(2x)^4$ as $2x^4$ rather than $16x^4$. Most textbooks will provide many examples for learners to practice. This link shows the above process and relates it to the binomial theorem showing correct notation. Go to https://www.mathsisfun.com then search for 'Binomial theorem'. A useful PowerPoint demonstration of Pascal's triangle and the formula for the binomial expansion using combinations is here (log in for free download): https://www.tes.co.uk/teaching-resource/binomial-expansion-powerpoint-6071493. You could use this in class, encouraging learners to work through the examples along the way. Alternatively, you could introduce learners to the formula and explain the notation: $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots b^n$ where $\binom{n}{r} = \frac{n!}{(n-r)!r!}$. Show learners that their calculator will work out these coefficients using the ${}^nC_r$ button and draw their attention to the patterns formed by these and the powers of *a* and *b*. If you prefer, you could consider combinations of terms to derive the result more formally. Most A Level textbooks have practice exercises on using the formula both to expand fully and to identify specific terms. Further development includes using an expansion to solve a problem such as: - Find the term in x^2 in the expansion of $(2+x)(3+4x)^5$ - Find the term independent of x in the expansion of $\left(2x^3 - \frac{1}{x}\right)^8$

Learning objectives	Suggested teaching activities
	Using such questions, you can encourage learners to consider the patterns formed when they expand binomial expressions. The link below (log in for free download) leads to a worksheet containing examples of some of these types of question. https://www.tes.co.uk/teaching-resource/a-level-maths-c2-binomial-expansion-worksheets-6146793 **(I)** **Past papers: (I)(F)** June 2014 paper 11, question 4 June 2014 paper 12, question 2 June 2014 paper 13, question 1 November 2014 paper 11, question 1 November 2014 paper 12, question 3 November 2014 paper 13, question 1 June 2013paper 11, question 2 June 2013 paper 12, question 2 June 2013 paper 13, question 4
Recognise arithmetic and geometric progressions.	You could start by giving learners a selection of sequences and asking them to identify any patterns. There are many examples in textbooks and on the internet (try searching for 'sequences + worksheet'). Help them to identify the main features of arithmetic and geometric progressions. It is a good idea to explain the meaning of the terms 'sequence', 'series' and 'progression'.
Use the formulae for the *n*th term and for the sum of the first *n* terms to solve problems involving arithmetic or geometric progressions.	For arithmetic progressions, introduce the notation that is commonly used for the first term, the common difference and the sum of *n* terms. Go to http://www.mathsisfun.com/ and search for 'Arithmetic sequences' to find the resource 'Arithmetic Sequences and Sums'. This a good example of the approach you could take with the whole class or with individuals wanting extra practice or revision. You can encourage learners to work out the formula for the *n*th term and then to derive the formula for the sum of *n* terms. For geometric progressions, introduce the notation that is commonly used for the first term, the common ratio and the sum of *n* terms. Encourage learners to work out the formula for the nth term themselves. You could give them a hint and encourage them to work out the formula for the sum of a geometric progression. Go to http://www.mathsisfun.com/ and search for 'Geometric sequences' to find the resource 'Geometric Sequences and Sums'. This is a good example of the approach that you could use, together with some examples. You could use this with the whole class or with individual learners wanting extra practice, revision or consolidation. Show learners that there are two possible versions of the formula for the sum of *n* terms of a geometric progression; the value of the common ratio can make one version easier to work with than the other.

Learning objectives	Suggested teaching activities
	To encourage independent learning, you could give learners the link to https://nrich.maths.org and ask them to search for ‘Summing Geometric Progressions’. The link below leads to an interactive site where learners can practise applying geometric progressions to everyday situations. https://www.khanacademy.org/math/precalculus/seq_induction/geometric-sequence-series/e/geometric-series **(I)** Most textbooks will have exercises for learners to practise using all the formulae, together with general problems. **(I)** **Past papers: (I)(F)** June 2014 paper11, question 4 (arithmetic) June 2014 paper 12, question 6 (both) June 2014 paper 13, question 2(ii) (both) November 2014 paper 11, question 7(ii) (arithmetic) November 2014 paper 12, question 8(a) (arithmetic) June 2013 paper 12, question 10 (both) June 2013 paper 13, question 9 (both)
Use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression.	You could start by considering a simple convergent geometric progression e.g. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.....$ Find the sum of *n* terms using the formula i.e. $S_n=\dfrac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}$ which leads to $S_n=2\left(1-\left(\frac{1}{2}\right)^n\right)$. You could illustrate this with two identical pieces of coloured paper. Stick one piece on the board (area 1), then cut the other piece in half and stick one piece to the board (area 1/2), cut the remaining piece in half and stick it to the board (area 1/4), and so on. By positioning the pieces carefully, learners can see that the sum can never be greater than 2. You can then draw the parallel with $n\to\infty$ and hence the sum to infinity, introducing the common notation used**.** As a group exercise, ask learners to see if they can deduce the general formula for the sum to infinity, together with any restrictions on the value of the common ratio. Most textbooks will have examples that will provide practice.

Learning objectives	Suggested teaching activities
	Past papers: (I)(F) June 2014 paper 13, question 2(i) November 2014 paper 11, question 7(i) November 2014 paper 12, question 8(b) November 2014 paper 13, question 4 (all) June 2013 paper 11, question 4

8. Differentiation

Learning objectives	Suggested teaching activities
Understand the idea of the gradient of a curve, and use the notations f'(x), f''(x), $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ (the technique of differentiation from first principles is not required).	Although the technique of differentiation from first principles is not a syllabus requirement, it is a good idea to show learners some of these principles to give them an appreciation of the basics of calculus, for example: http://wwwf.imperial.ac.uk/metric/metric_public/differentiation/ The link below leads to three files you could also use (log in for free download), together with a worksheet which would give learners the idea of limits and finding gradients of tangents to curves. You could use these as extension work for able learners. https://www.tes.co.uk/teaching-resource/differentiation-from-first-principles-6147220
Use the derivative of x^n (for any rational n), together with constant multiples, sums, differences of functions, and of composite functions using the chain rule.	The link below provides worksheets of varying types which will give learners practice at basic differentiation. There is also a dominoes game which you could use to check learning (log in for free download): https://www.tes.co.uk/teaching-resource/a-level-maths-c1-differentiation-worksheets-6146718 **(I)** First ensure that learners are confident in the differentiation of constant multiples, sums and differences before introducing the chain rule for differentiating a composite function (function of a function). Textbooks will have many exercises of examples for learners to practice. **(I)** There are useful resources on the chain rule at this link (log in for free download): https://www.tes.co.uk/teaching-resource/chain-rule-6146849 **(I)**
Apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change (including connected rates of change).	You should remind learners that the gradient function will give the gradient of a **tangent** to the curve at a particular point, so differentiation will be the first step in finding the equation of the tangent. This Java applet uses a man surfing to illustrate the situation http://www.ies-math.com/math/java/calc/doukan/doukan.html Work through a straightforward example on the board or give it out as a group exercise, e.g. Find the equation of the tangent to curve $y = 2x^3 - 4x^2 + 5x - 7$ at the point where $x = 1$. The activity could be extended to finding the equation of the normal at that point and some more examples of increasing difficulty. Learners should be able to progress to textbook examples of different types. **(I)** The link below provides further practice at tangents and normals using an interactive approach.

Learning objectives	Suggested teaching activities
	https://www.khanacademy.org/math/differential-calculus/derivative_applications/normal-tangent-line-eq/e/applications-of-derivatives--tangent-and-normal-lines The link below provides further practice together with examples where learners are expected to use the equations they have just found. https://www.tes.co.uk/teaching-resource/maths-worksheets-tangents-and-normals-6139694 **(I)** Define increasing and decreasing functions by using the example of a cubic function and asking learners to imagine walking along it from left to right (you may wish to lay a piece of rope or heavy string on the floor in the shape of a cubic curve. Ask one learner to walk slowly along it while the rest of the class describes the gradient at each stage). The link below provides two files, one of which is a general worksheet and the other is a matching game. https://www.tes.co.uk/teaching-resource/a-level-maths-c1-worksheet-function-turning-point-6146765 **(I)** Introduce rates of change by reminding learners that $\frac{\mathrm{d}y}{\mathrm{d}x}$ represents the rate of change of y with respect to x. Introduce different derivatives e.g. $\frac{\mathrm{d}A}{\mathrm{d}t}$, so this must be the rate of change of A with respect to t. Then introduce the idea that t could represent time. Remind learners of the chain rule from the previous section and show how they can form equations with it e.g. $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$. Introduce the idea that A could represent the area of a circle of radius r (you could give the example of a circular ink stain) so, if they know the rate of change of the radius with respect to time, they can calculate the corresponding rate of change of the area. Most textbooks will have exercises that learners may use for practice. **(I)(F)** **Past papers: (I)(F)** June 2014 paper 11, question 4 June 2014 paper 12, question 9(i) and (ii) November 2014 paper 11, question 9(i) November 2014 paper 12, question 4

Learning objectives	Suggested teaching activities
	June 2013 paper 13, question 11(i)
Locate stationary points, and use information about stationary points in sketching graphs (the ability to distinguish between maximum points and minimum points is required, but identification of points of inflexion is not included).	This link below provides a worksheet which you could use to introduce stationary points: http://www.nuffieldfoundation.org/ Search for 'Calculus', click on 'Level 3 Calculus' then find resources on Stationary Points. Points of inflexion are mentioned but these parts could be omitted or used as extension material for the more able learner. The nature of the stationary points is also covered. There are worked examples and further examples for learners to try themselves. **(I)** The link below also provides a quick overview of the topic that learners could use for revision or consolidation. http://www.bbc.co.uk/bitesize/higher/maths/calculus/differentiation2/revision/1/ Most textbooks have plenty of examples on stationary points for learners to practise. **(I)(F)** Highlight the main features needed when sketching curves: find any intercepts with the coordinate axes, find any stationary points and determine their nature then attempt to sketch a curve. Questions are often structured so that learners identify the main features before they are asked to sketch the graph. **Past papers: (I)(F)** June 2014 paper 11, question 12 (involves some integration) June 2014 paper 12, question 8 (involves some integration) June 2014 paper 13, question 9 (an application question) November 2014 paper 11, question 9(iii) November 2014 paper 12, question 10 (involves some integration) November 2014 paper 13, question 8(i) June 2013 paper 11, question 9(i), (ii)

9. Integration

Learning objectives	Suggested teaching activities
Understand integration as the reverse process of differentiation, and integrate $(ax + b)^n$ (for any rational *n* except –1), together with constant multiples, sums and differences. .	Start with an example of a gradient function e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2+2$ and ask learners to deduce possible functions that could have this gradient. You can then introduce the idea of including a constant of integration to cover an infinite number of possible solutions. Graphically, learners could visualise this as a family of curves; knowing the constant of integration will identify which curve they need. Starting with indefinite integration will encourage learners to include a constant of integration in their answers. You may wish to introduce notation here. Explain the importance of using it correctly; many learners forget to include $'\mathrm{d}x'$ if they do not fully understand what it means. Integrate various examples of functions, leaving $(ax+b)^n$ until the end. Ask learners to work out an integral of this type by expanding it first, e.g. $\int(2x+3)^3\,\mathrm{d}x$, then they can see if they can deduce a general formula for integrating this type of expression. They can check it works by differentiating again. You could also ask them why this formula will work for any rational *n* except –1. This link leads to a set of PowerPoint presentations. Lesson 1 provides a good approach to the introduction of integration. https://www.tes.co.uk/teaching-resource/integration-powerpoint-6402321 The link below leads to a set of four files. The file 'Introduction to Integration' provides a short exercise for learners. https://www.tes.co.uk/teaching-resource/indefinite-integration-6146782
Solve problems involving the evaluation of a constant of integration, e.g. to find the equation of the curve through (1, –2) for which $\frac{\mathrm{d}y}{\mathrm{d}x}=2x+1$.	Point out to learners that they are now going to be evaluating the constant of integration. This link leads to a set of PowerPoint presentations. Lesson 2 provides a good approach to the evaluation of a constant of integration. https://www.tes.co.uk/teaching-resource/integration-powerpoint-6402321 The link below leads to a set of four files. The files 'Finding the Arbitrary Constant' and 'Indefinite Integration Problems' provide good resources for learners to use in or out of the classroom. The file 'Indefinite Integration Dominoes' provides a group activity which you could use to check learning either at the start or the end of a lesson. https://www.tes.co.uk/teaching-resource/indefinite-integration-6146782 **(I)**

Learning objectives	Suggested teaching activities
	Past papers: (I)(F) June 2014, paper 11, question 12 (i) June 2014 paper 13, question 6 June 2013 paper 12, question 1 June 2013 paper 13, question 1
Evaluate definite integrals (including simple cases of 'improper' integrals, such as $\int_0^1 x^{-\frac{1}{2}}\mathrm{d}x$ and $\int_1^\infty x^{-2}\mathrm{d}x$).	Introduce learners to the notation used for definite integration and demonstrate how to evaluate an integral using limits and square brackets. Some teachers may prefer to introduce the area of a region enclosed by a curve, the *x*-axis and lines $x = a$ and $x = b$ at this point. The link below leads to a set of PowerPoint presentations. Part of lesson 3 provides a quick overview on evaluating definite integrals. https://www.tes.co.uk/teaching-resource/integration-powerpoint-6402321 Some learners may be daunted by the idea of an infinite limit, so it will be useful to work through a couple of examples, such as the one in this syllabus section. Looking at the graph may also help them. The link below leads to a set of four files. The file 'Definite Integration Intro' provides good examples of the notation used and evaluation of definite integrals. The file 'Definite Integration Dominoes' provides a group activity which you could to check learning either at the start or the end of a lesson. https://www.tes.co.uk/teaching-resource/a-level-maths-c2-definite-integration-worksheet-6146778
Use definite integration to find: - the area of a region bounded by a curve and lines parallel to the axes, or between two curves - a volume of revolution about one of the axes.	You could introduce this topic by considering the area under a curve divided into a series of strips. There are various approaches to this e.g. looking at (1) the area of a set of strips containing the area required and (2) the area of a set of strips which is part of the area required. The value of the area required must therefore lie between the values of areas (1) and (2). You could make a comparison with the actual value obtained by integration. Alternatively, you could demonstrate a formal proof to those learners who would appreciate it, or you could go directly to the statement that the area of the region enclosed by a curve, the *x*-axis and lines $x = a$ and $x = b$ is $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$, where $y = \mathrm{f}(x)$ is the equation of the curve. One possible approach is at https://www.mathsisfun.com/ . Search for 'Definite integrals'. Make sure you include an example of a curve that passes below the *x*-axis so that learners realise the effect it has on

Learning objectives	Suggested teaching activities
	the value of their integral e.g. the area enclosed by the curve $y=(x-3)^2(x+1)$ and the *x*-axis. The link below leads to a set of four files. The files 'Using Integration to Find Areas' and 'Definite Integration to find Areas' provide a good approach to the introduction of the topic together with examples which may be used as practice. https://www.tes.co.uk/teaching-resource/a-level-maths-c2-definite-integration-worksheet-6146778 **(I)** To find the area enclosed by two curves, show learners that they can integrate the difference between the two functions $\int_a^b (y_1 - y_2)\,\mathrm{d}x$ where $x=a$ and $x=b$ are the *x*-coordinates of the points of intersection of the two curves. The video in the link below shows this method. **(I)** https://www.khanacademy.org/math/integral-calculus/solid_revolution_topic/area-between-curves/v/area-between-curves For volumes of revolution, you could use an approach like the one in the link below. It involves splitting the area under the curve into strips which, when rotated, form discs. This leads to the formula for rotation about the ***x*-axis**. You could challenge learners to deduce the formula required when the area is rotated about the ***y*-axis** instead. https://www.khanacademy.org/math/integral-calculus/solid_revolution_topic/disc-method/v/disk-method-around-x-axis The link below leads to a PowerPoint presentation on the disc method, together with a worksheet of examples which learners can use for practice. https://www.tes.co.uk/teaching-resource/c4-integration--volume-of-revolution-6340006 **(I)** **Past papers: (I)(F)** June 2014 paper 12, question 9(iii) June 2014 paper 13, question 10 November 2014 paper 11, question 11 November 2014 paper 12, question 1 November 2014 paper 13, question 9 June 2013 paper 11, question 10 June 2013 paper 12, question 11 June 2013 paper 13, question 11

Cambridge International Examinations
1 Hills Road, Cambridge, CB1 2EU, United Kingdom
tel: +44 1223 553554   fax: +44 1223 553558
email: info@cie.org.uk   www.cie.org.uk

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CAMBRIDGE ASSESSMENT