991 ES

SC1100412

525023

Centre Number	Candidate Number
Syllabus Title: MATHEMATICS	
Syllabus Number: 9709	Paper Number: 32
Name (in Block Letters): **Script 2 Paper 32**	
Signature:	
Date: 19th May - 2015	

Answer Book

Instructions to Candidates

1. Insert the information required in the spaces on the left.
2. Use both sides of the paper.
3. Write the question number in the left-hand margin.
4. Rule a line across the page after each question.
5. Do all the rough work in this answer book and cross it through, without making it illegible. No part of this book is to be torn out.
6. Check that you have inserted the information required on each supplementary sheet.

Q	M
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
T	

UNIVERSITY of CAMBRIDGE
International Examinations

Question number

Write on both sides of the paper

Page

Q1- $\int_0^{1/2\pi} \ln(1+\sin x)\,dx$

$h = \frac{\frac{1}{2}\pi - 0}{3} = \frac{1}{6}\pi$

$= \frac{h}{2}\left(f(a) + 2f(a+h) + 2f(a+2h) + f(b)\right)$

$= \frac{\pi/6}{2}\left(f(0) + 2f(\pi/6) + 2f(\pi/3) + f(\frac{1}{2}\pi)\right)$

$= \frac{1}{12}\pi\left(0 + 0.811 + 1.247 + 0.230\right)$

$= 0.60 \text{ unit}^2$

Q2- $u = 4^x$

$u + 4^2 = 4^{x+2}$

~~$u = 4^{x+2} - 4^2$~~

~~$u =$~~

$u + 4^2 = u^3$

$u - u^3 = 4^2$

$u(1 - u^2) = 4^2$

$u = 4^2$

$4^x = 4^2$

$x = 2$

$x\ln 4 + 2\ln 4 = x + 2\ln 4$

$\ln 4(x+2) = \ln(4)(x+2)$

on extra sheet

$1 - 4^{2x} = 4^2$

$\ln\left(\frac{1}{4^{2x}}\right) = \ln e^{4^2}$

$\frac{1}{4^{2x}} =$

Q3- $y = \cos x \cos 2x$

$u = \cos x \quad u' = -\sin x$

$v = \cos 2x \quad v' = -2\sin 2x$

$\frac{dy}{dx} = uv' + vu'$

$0 = (\cos x)(-\sin x) + (\cos 2x)(-2\sin 2x)$

~~$0 = -\sin x (\cos x +$~~

~~$0 = \cos x (-\sin x) + \cos 2x$~~

~~$0 = \cos x (-\sin$~~ $0 = -\sin x (\cos x + \cos 2x - 2\sin x$

$0 = \cos x + \cos 2x (-2\sin x \cos x)$

~~$0 = -\sin x$~~

$0 = -\cos x \sin x - \cos 2x \sin 2x$

$= -\cos x ($ ~~$\sin x - \cos x \sin 2x$~~ $)$

$0 = -\cos x \sin x$

$1 - \cos x \sin x = 0$

$\cos x \sin x = -1$

on extra sheet

Question number

Write on both sides of the paper

Page

Leave blank

Q4 i- $3\sin\theta + 2\cos\theta = R(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$

$3 = R\cos\alpha$ $\qquad$ $2 = R\sin\alpha$

$\frac{3}{R} = \cos\alpha$ $\qquad$ $\frac{2}{R} = \sin\alpha$

$Hyp^2 = P^2 + b^2$

$\sqrt{Hyp^2} = \sqrt{(3)^2 + (2)^2}$

$R = \sqrt{13}$

$\frac{2}{\sqrt{13}} = \sin\alpha$

$\sin^{-1}\left(\frac{2}{\sqrt{13}}\right) = \alpha$

$33.69^\circ = \alpha$

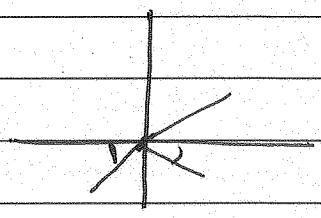
ii $\sqrt{13}\sin(\theta + 33.69) = 1$

$\theta + 33.69 = \sin^{-1}\left(\frac{1}{\sqrt{13}}\right)$

$\theta + 33.69 = 16.1$

$\theta = -17.59$

$[97.6, 342.4)$



Q5- Circumference = $2\pi r$

$S = r\theta$

$= 2\pi r$

Shaded region perimeter

$\tan\theta r + \tan\theta r + 2\pi r = 2\pi r$

$2\tan\theta r + 2\pi r = 2\pi r$

$2r(\tan\theta + \pi) = 2\pi r$

$\tan\theta = \pi - x$

$\tan x = \pi - x$ → shown.

ii- $f(x) = \pi - x - \tan x$

$f(1) = \pi - 1 - \tan(1)$

$= 0.584$

$f(1.3) = \pi - 1.3 - \tan(1.3)$

$= -1.76$

root lies between these two points because the sign is changed.

iii - $x_{n+1} = \tan^{-1}(\pi - x)$

$x_1 = 1$

1.1339

1.0643

1.1221 root = 1.11

1.1110

1.1132

Question number

Write on both sides of the paper

Page

Leave blank

Q6- $I = \int_0^1 \frac{\sqrt{x}}{2-\sqrt{x}}$

$u = 2 - \sqrt{x}$

$(\sqrt{x})^2 = (2-u)^2$

$x = (2-u)^2$

new limits $u = 2 - \sqrt{x} = 1$

new limits $2 - \sqrt{0} = 2$

$\frac{dx}{du} = -2(2-u)$

$dx = -2(2-u)\,du$

$\int_1^2 \frac{2-u}{2-(2-u)} \cdot -2(2-u)\,du$

$\int_1^2 \frac{2(2-u)^2}{2-2+u}\,du$

$\int_1^2 \frac{2(2-u)^2}{u}\,du$ shown.

ii $\frac{2(2-u)^{2+1}}{(u)(-1)(2+1)} \to x$

$= \frac{2(2-u)^3}{-3u}$

$= \left| \frac{2(2-u)^3}{-3u} \right|_1^2$

Write on both sides of the paper

Question number

Page

Leave blank

ii - $\int_1^2 \frac{2(2-u)^2}{u} \, du$ $\quad \frac{2}{-4(2-u)}$

$\left| -4(2-u)\ln(u) \right|_1^2$

$\left| +4(2-0)\ln(2) \right| - \left| -4(2-u)\cdot\ln(1) \right|$

$\left| 8\ln 2 \right| - \left| +4+1 \right|$

$8\ln 2 - 5$ shown

Q7-

$$\left(\sqrt{-1+4\sqrt{3}i}\right)^2 = (x+iy)^2$$

$$-1+4\sqrt{3}i = (x+iy)(x+iy)$$

$$-1+4\sqrt{3}i = x^2+2iy+i^2y^2$$

$$-1+4\sqrt{3}i = x^2+2ixy-y^2$$

$$-1 = x^2-y^2 \quad \text{---} \quad (1)$$

~~$4\sqrt{3}i = 2ixy$~~ ~~$\sqrt{3} = 1$~~ ~~2~~ ~~y~~ ~~$2\sqrt{3} = y$~~ ~~---~~ ~~(11)~~

$$4\sqrt{3}i = 2ixy \quad \text{---} \quad (11)$$

$$\frac{2\sqrt{3}}{x} = y$$

$$\text{---} \quad (111)$$

~~$-1 = x^2-(2\sqrt{3})^2$~~

~~$-1+12 = x^2$~~

~~$\sqrt{11} = \sqrt{x^2}$~~

~~$\sqrt{11} = x$~~

$$-1 = x^2-\left(\frac{2\sqrt{3}}{x}\right)^2$$

$$-1 = x^2-\frac{12}{x^2}$$

$$-1 = \frac{x^4-12}{x^2}$$

$$-x^2 = x^4-12$$

$$x^4+x^2-12=0$$

$$x^2+4x^2-3x^2-12=0$$

$$x(x+4)-3(x+4)=0$$

$$(x-3)(x+4)=0$$

$$x=3 \qquad x=-4$$

Write on both sides of the paper

Question number

Page

Leave blank

$-1 = (3)^2 - y^2$

$-1 - 4 = -y^2$

$-5 = -y^2$

$\sqrt{5} = \sqrt{y^2}$

$\sqrt{5} = y$

$-1 = (-4)^2 - y^2$

$-1 - 16 = -y^2$

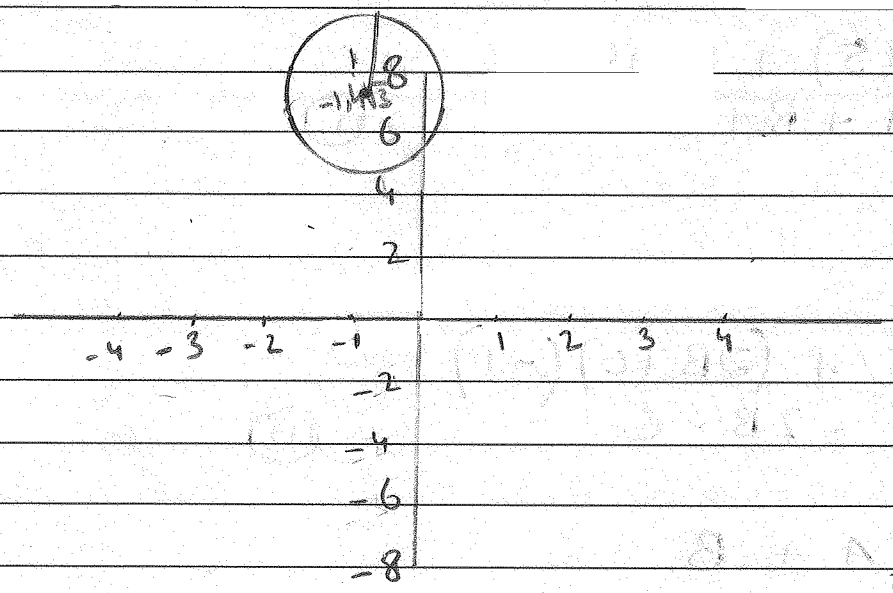
$\sqrt{17} = \sqrt{y^2}$

$\sqrt{17} = y$

roots

$3 + \sqrt{5}i$, $-4 + \sqrt{17}i$)

ii-



$z - 4 = 1$

$|z - (-1 + 4\sqrt{3}i)| = 1$

z

arg of z

$\theta = \tan^{-1}\left[-\frac{4\sqrt{3}}{1}\right]$

$\theta = \tan^{-1} -4\sqrt{3}$

Q8- $\frac{5x^2+x+6}{(3-2x)(x^2+4)} = \frac{A}{3-2x} + \frac{Bx+C}{x^2+4}$

$5x^2+x+6 = A(x^2+4)+(Bx+C)(3-2x)$

$x=0$

$6 = A(4)+(C)(3)$

$6 = 4A+3C$ — (I)

$x=1$

$5+1+6 = A(5)+(B+C)(1)$

$12 = 5A+B+C$ — (II)

$x=2$

$28 = A16 + (2B+C)(-1)$

$28 = 16A - 2B - C$ — (III)

$C = 12 - 5A + B$

$28 = 16A - 2B - (12 - 5A + B)$

$28 = 16A - 2B - 12 - 5A - B$

$40 = 11A - 3B$

$x=3$

$12 - 5A + B$

$\frac{-B-12}{-5} = A$

$54 = 13A + (3B+C)(-3$

$54 = 13A + 9B - 3C$

$x = 3$

$54 = 13A - 9B - 3(12 - 5A + B)$

$54 = 13A - 9B - 36 + 15A - 3B$

$90 = 28A - 12B$

$\frac{40 + 3B}{11} = A$

$90 = 28\left(\frac{40 + 3B}{11}\right) - 12B$

$90 = \frac{1120 + 84B - 132B}{11}$

$990 = 1120 - 48B$

$-130 = -48B$

$\frac{65}{24} = B$

$x = -2$

$24 = 8A + (-2B + C)(-1)$

$24 = 8A + 2B - C$ — (1)

$x = 1$

$12 = 5A + B + C$ — (I)

$x = 2$

$28 = 16A - 2B - C$ — (II)

$x = -2$

$24 = 8A + 2B - C$ — (III)

$C = 8A + 2B - 24$

$12 = 5A + B + 8A + 2B - 24$

$12 = 13A + 3B - 24$

$36 = 16A$

$\frac{9}{4} = A$

~~$28 = 16\left(\frac{9}{4}\right) - 2B = 8A + 2B - 24$~~

~~$28 = 144 - 2B$~~

$28 = 16\left(\frac{9}{4}\right) - 2B - 18 - 2B + 24$

$28 = 36 - 4B + 6$

$-14 = -4B$

$\frac{7}{2} = B$

991 ES

446472

ADDITIONAL

Answer Book

Centre Number	Candidate Number
Syllabus Title: MATHEMATICS	
Syllabus Number: 9709	Paper Number: 32
Name (in Block Letters)	
Signature	
Date: 19th May 2015	

Instructions to Candidates

1. Insert the information required in the spaces on the left.
2. Use both sides of the paper.
3. Write the question number in the left-hand margin.
4. Rule a line across the page after each question.
5. Do all the rough work in this answer book and cross it through, without making it illegible. No part of this book is to be torn out.
6. Check that you have inserted the information required on each supplementary sheet.

UNIVERSITY of CAMBRIDGE
International Examinations

Question number

Write on both sides of the paper

Page

Leave blank

Q9- $\frac{dx}{dt} = \frac{xe^{-t}}{k+e^{-t}}$

$\int \frac{1}{x} dx = \int \frac{e^{-t}}{k+e^{-t}} dt$

$\int \frac{1}{x} dx = \int \frac{e^{-t}}{k+e^{-t}} dt$

$\ln|x| = -\ln|k+e^{-t}|$

$x = 10 \quad t = 0$

$\ln|10| = -\ln|k+e^{-0}|$

$\ln|10| = -\ln|k+1|$

$10 = -k-1$

$10+1 = -k$

$-11 = k$

ii- $\ln|x| = -\ln|k+\frac{1}{e^t}|$

$\ln|20| = -\ln|k+\frac{1}{e^2}|$

$20 = -k - \frac{1}{e^2}$

$e^2(20) = -e^2k - 1$

$e = -2k - 1$

$1 - \frac{2}{e} = -k$

iii- $\overrightarrow{OA} = 2i - j + 3k \qquad \overrightarrow{OB} = i + j + 5k$

$L = i + j + 2k + \mu(3i + j - k)$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$

$r = 2i - j + 3k + s(-i + 2j - 2k)$

$\begin{bmatrix} 1 + 3\lambda \\ 1 + \lambda \\ 2 - \lambda \end{bmatrix} = \begin{bmatrix} 2 - s \\ -1 + 2s \\ 3 - 2s \end{bmatrix}$

$1 + 3\lambda - 2 = -s$

$1 - 3\lambda = s$

$1 + \lambda = -1 + 2(1 - 3\lambda)$

$1 + \lambda = -1 + 2 - 6\lambda$

$1 + \lambda = 1 - 6\lambda$

$\lambda + 6\lambda = 0$

$7\lambda = 0$

$\lambda = 0$

$1 = -1 + 2s$

$\frac{2}{2} = s$

$1 = s$

$2 - 0 = 3 - 2$

$2 \neq 1$

L does not intersect

$$n = \begin{vmatrix} i & j & k \\ 3 & 1 & -1 \\ -1 & 2 & -2 \end{vmatrix}$$

$$= i\begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} - j\begin{vmatrix} 3 & -1 \\ -1 & -2 \end{vmatrix} + k\begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix}$$

$$= i|2+2| - j|-6-1| + k|6+1|$$

$$= 4i + 7j + 7k$$

$$r \cdot n = a \cdot n$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 7 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 7 \\ 7 \end{pmatrix}$$

$$4x + 7y + 7z = 8 - 7 + 21$$

$$4x + 7y + 7z = 22$$

991 ES

446372

ADDITIONAL

Answer Book

Centre Number	Candidate Number
Syllabus Title: MATHEMATICS	
Syllabus Number: 9709	Paper Number: 32
Name (in Block Letters)	
Signature	
Date: 19th - May - 2015	

Instructions to Candidates

1. Insert the information required in the spaces on the left.
2. Use both sides of the paper.
3. Write the question number in the left-hand margin.
4. Rule a line across the page after each question.
5. Do all the rough work in this answer book and cross it through, without making it illegible. No part of this book is to be torn out.
6. Check that you have inserted the information required on each supplementary sheet.

UNIVERSITY of CAMBRIDGE
International Examinations

Write on both sides of the paper

Question number | Page | Leave blank

Q2- $4^x + 4^2 = 4^{x+2}$

$u + 4^2 = 4^{x+2}$

$u = 4^{x+2} - 4^2$

$\ln u = x \ln 4$

$\ln u = \ln 4x$

$\ln u = \ln \left(\frac{4^{x+2}}{4^2} \right)$

$\ln 4x = \ln \left(\frac{4^{x+2}}{4} \right)$

$4^x = 4^{x+2}$

$4^{2x} = 4^{x+2}$

$2x = x + 2$

$2x - x = 2$

$x = 2$

Q3- $y = \cos x \cos 2x$

$u = \cos x \quad u' = -\sin x$

$v = \cos 2x \quad v' = -2\sin 2x$

$\frac{dy}{dx} = uv' + vu'$

$= -2\sin 2x (\cos x) + \cos 2x (-\sin x)$

$= -2\sin 2x \cos x - \sin x \cos 2x$

~~$= -2\sin$~~

$= -\sin x (2\sin x \cos x - \cos 2x)$

$0 = -\sin x$

$x = 0$

$(2\sin x \cos x - \cos 2x) = 0$

$\cos x (2\sin x - \cos x) = 0$

$\cos x = 0$

$x = 90$

$2\sin x = \cos x$

$\frac{\sin x}{\cos x} = \frac{1}{2}$

$\tan x = \frac{1}{2}$

$x = \tan^{-1}\left(\frac{1}{2}\right)$

$x = 26.6$

Q8 continue

$c = 8\left(\frac{9}{4}\right) + 2\left(\frac{7}{2}\right) - 24$

$= 18 + 7 - 24$

$c = 1$

$\frac{5x^2 + x + 6}{(3-2x)(x^2+4)} = \frac{9}{4(3-2x)} + \frac{\frac{7}{2}x + 1}{x^2+4}$

$= \frac{9}{4(3-2x)} + \frac{7x+2}{2(x^2+4)}$

Q8 ii - $\frac{9}{4(3-2x)} + \frac{7x+2}{2(x^2+4)}$

$\frac{9}{(4)(3)(1-\frac{2}{3}x)}$

$(1-\frac{2}{3}x)^{-1}$

$(1)^{-1} + (1)^{-1-1}(-1)(-\frac{2}{3}x) + \frac{(-1)(-1-1)}{2!}(-\frac{2}{3}x^2)$

$1 + \frac{2}{3}x + \frac{4}{9}x^2$

$\frac{9}{12}(1+\frac{2}{3}x+\frac{4}{9}x^2)$

$\frac{9}{12} + \frac{1}{2}x + \frac{1}{3}x^2 + x^2$

$\frac{9}{12} + \frac{1}{2}x + \frac{4}{3}x^2$

$\frac{7x+2}{2}(x^2+4)^{-1}$

$(7x+2)(\frac{1}{2})$

$(14x+4)(1-\frac{x^2}{4})$

$-\frac{4x^2}{4}$

$(\frac{x^2}{4}+1)^{-1}$

$(1)^{-1} + (1)(-1)(\frac{x^2}{4})$

$1 - \frac{x^2}{4}$