SC1100397

12 PAGE ANSWER BOOKS

GCSE, GCE, VCE and GNVQ Examining Bodies

Examining body	CIE
Centre number	Candidate number
Subject/module title	Mathametics.
Paper reference	9709 / 32
Surname	**Script 1, Paper 32**
Other names	
Candidate signature	

For examiner's use

Examiner's initials

Question number	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

- Use blue or black ink or ball-point pen.
- Write the information required in the spaces above.
- Use both sides of the paper.
- Write the question number in the left-hand margin.
- Rule a line across the page after each answer.
- Do all your rough work in this answer book and cross through any work you do not want marked. Do not tear out any part of this book. All work must be handed in.
- Write the numbers of the questions you answer in the order attempted in the left-hand column of the boxes opposite.
- Check that you have written the information required on each additional sheet used and have attached each sheet to this book.

Write here how many additional sheets you have used (if any).

Question number | Leave blank

Q1. $h = \dfrac{\pi/2}{6} = \dfrac{\pi}{6}$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	0	0.405	0.624	0.693

$\text{Area} = \frac{1}{2} \times \frac{\pi}{6}(0 + 0.693) + \frac{\pi}{6}(0.405 + 0.624)$

$= 0.720$ (Ans)

AB12

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Q2 ~~[illegible]~~ of $u = 4^x$,

$4^x + 4^2 = 4^{x+2}$

$u + 16 = 16u$

$15u = 16$

$u = \frac{16}{15}$

$\therefore \quad 4^x = \frac{16}{15}$

$x\ln 4 = \ln\frac{16}{15}$

$x = 0.0466$ (Ans)

Q3 $y = \cos x \cos 2x$.

~~$\frac{dy}{dx} = \cos x \cdot 2(-\sin 2x) - \cos 2x \sin x$~~

~~$= -2\cos x \sin 2x - \cos 2x \sin x$~~ $= 0$.

$= y = \cos x(\cos^2 x - \sin^2 x)$

$= \cos x(\cos^2 x - 1 + \cos^2 x)$

$= 2\cos^3 x - \cos x$

$\frac{dy}{dx} = 4\cos^2 x(-\sin x) + \sin x = 0$.

$\sin x(1 - 4\cos^2 x) = 0$.

$\sin x = 0$. $\quad 1 - 4\cos^2 x = 0$.

$x = 0$ $\quad \cos x = \sqrt{\frac{1}{4}}$

$\downarrow$ $\quad x = \cos^{-1} 0.5$

Not in range $\quad x = \frac{\pi}{3}$

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Ans : $x = \pi/3$

4 i) $3\sin\theta + 2\cos\theta = R\sin(\theta+\alpha)$.

$R = \sqrt{3^2+2^2} = \sqrt{13}$.

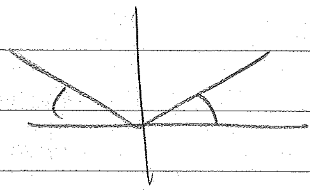
$\alpha = \tan^{-1}\frac{2}{3} = 33.69^\circ$.

$\sqrt{13}\sin(\theta + 33.69^\circ)$ (Ans)

(ii) $3\sin\theta + 2\cos\theta = 1$.

substituting with sol of (i),

$\sqrt{13}\sin(\theta + 33.69) = 1$.



$\theta + 33.69 = \sin^{-1} 1/\sqrt{13}$

'' $\theta = 16.10, 163.90, 376.10$.

$\therefore \theta = -17.59^\circ, 130.21, 342.41$

Ans: $\theta = 130.21^\circ, 342.41^\circ$

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5i) Show $\tan x = \pi - x$.

Circumference $= 2rx + 2r\tan x$

Shaded region $= \cancel{\frac{1}{2}r^2 \sin 2x - \frac{1}{2}(2x)^2 r}$

$= \left[\frac{1}{2} \times r \times AT\right] \times 2 - \frac{1}{2}(2x)^2 r$

$= r^2 \tan x - 2x^2 r$ as $\tan x = \frac{AT}{r}$

$\therefore r\tan x - 2x^2 r = 2rx + 2r\tan x$

$\cancel{\tan x = 2r + 2x^2}$

$\tan x - 2x^2 = 2x + 2\tan x$

$\cancel{\tan x = -2x - 2x^2 = \tan x}$

$\tan x \quad -2x(1+x)$

$2r\tan x = \cancel{2rx} + 2\pi r - 2rx$

$\tan x = \pi - x$

shown.

(ii) $\tan x - \pi + x = 0$

~~if x = 1~~	~~= -2.58~~	if $x =$ 0.7	=	-1.6
~~1.1~~	~~2.28~~	1.0	=	-0.58
~~1.2~~	~~1.77~~	1.3	=	1.76
~~1.3~~	~~0.84~~			
~~1.4~~				

As there was a sign change between 1.0 & 1.3, root is in

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that range.

(iii) $x_{n+1} = \tan^{-1}(\pi - x_n)$

taking x_n as 1.15, $x_{n+1} = 1.1055$

~~xn~~.

" 1.1143

" 1.1126

" 1.1129

" 1.1128

" 1.1128

root = 1.11 (Ans)

6(i) $\int_0^1 \frac{\sqrt{x}}{2-\sqrt{x}}\,dx$

if $u = 2 - \sqrt{x}$.

$\frac{du}{dx} = -\frac{1}{2}x^{(-1/2)}$

~~$\int \frac{2-u}{u}\,dx$~~

~~$dx = \frac{1}{2\sqrt{x}}\,du$~~

$dx = \frac{2du}{x^{-1/2}}$.

$\frac{\sqrt{x}}{u} \times 2\sqrt{x}\,du$

As, $u = 2 - \sqrt{x}$

$\sqrt{x} = 2 - u$

$= \frac{2x}{u}\,du$

$x = (2-u)^2$

$\int_1^2 \frac{2(2-u)^2}{u}\,du$, Ab shown.

if $x = 1$ $u = 1$

$x = 0$ $u = 2$

(ii) $2\int \frac{4-4u+u^2}{4}$

$= 2\int \frac{4}{4} - 4 + u$

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$= 2(4\ln u) - 2(4u) + 2(u^2/2) \Big]_1^2.$

$8\ln u - 8u + u^2 \Big]_1^2.$

$(8\ln 2 - 16 + 4) - (-8+1)$

$8\ln 2 - 5$ // shown.

7(i) $u = -1 + 4\sqrt{3}\, i$

$\sqrt{-1+4\sqrt{3}i} = a + bi$

$-1 + 4\sqrt{3}i = (a+bi)^2$

$-1 + 4\sqrt{3}i = a^2 + 2abi - b^2$

$a^2 - b^2 = -1$ — (i)

$2ab = 4\sqrt{3}$

$ab = 2\sqrt{3}$

$b = \frac{2\sqrt{3}}{a}$ — (ii)

Substituting (ii) into (i),

$a^2 - \left(\frac{2\sqrt{3}}{a}\right)^2 = -1$

$\cancel{\frac{2\sqrt{3}}{2i}}$

$\frac{\sqrt{3} \times i}{i \times i}$

$-\sqrt{3}\, i$

$a^4 - 12 = -a^2$

$a^4 + a^2 - 12 = 0$

$(a^2 + 4)(a^2 - 3)$

$\cancel{a^2 = 4, -3}$

$\cancel{a = \pm 2, \pm\sqrt{3}i}$

$b = \cancel{\pm\sqrt{3}}, \mp 2i$

$a^2 = 3, -4$

$a = \pm\sqrt{3}, \pm 2i$

$b = \pm 2, \mp\sqrt{3}i$

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roots =

$\cancel{\pm(2+\sqrt{3}i)}$ ~~or~~ $\cancel{\sqrt{3}i - 2i}$

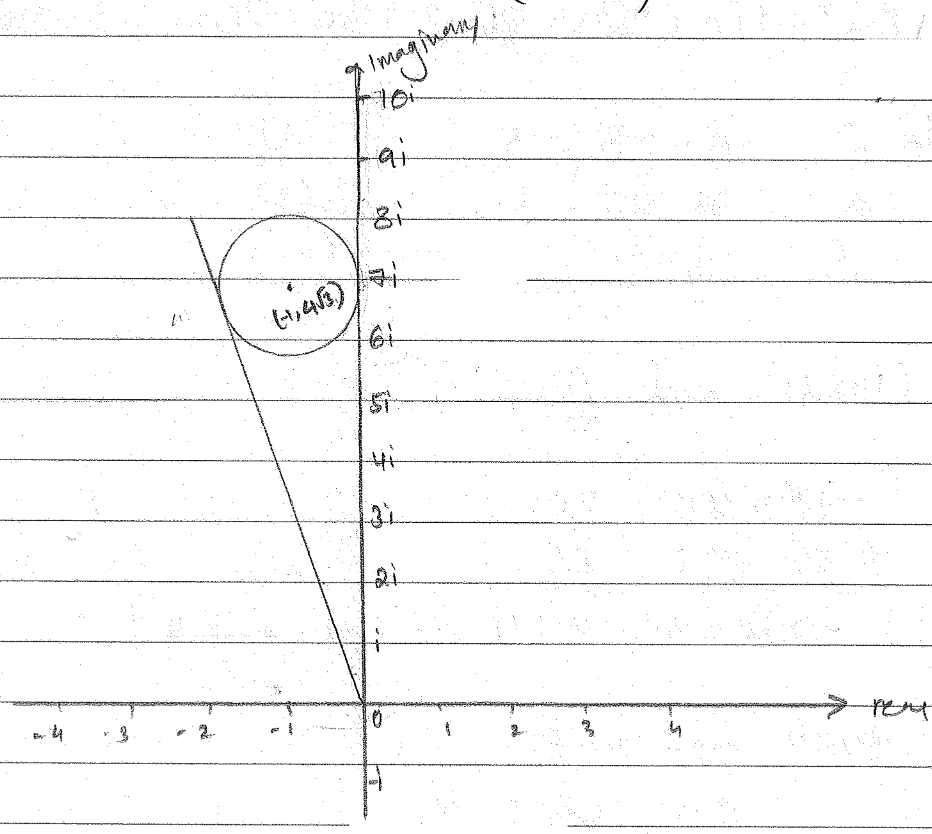
$\pm(\sqrt{3}+2i)$ ~~or~~ $\cancel{2i-(\sqrt{3}i \times i)}$

and $\pm\sqrt{3}-2i$ or $2i+(\sqrt{3}i)i$

$= 2i+\sqrt{3}$

So, Ans : $\pm(\sqrt{3}+2i)$ and $\pm(\sqrt{3}-2i)$

(ii)



Greatest argument = 110°

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8i) $\dfrac{5x^2+x+6}{(3-2x)(x^2+4)}$

$= \dfrac{A}{3-2x} + \dfrac{Bx+C}{x^2+4}$

$= A(x^2+4)+(Bx+C)(3-2x) = 5x^2+x+6$.

$Ax^2+4A+3Bx-2Bx^2+3C-2Cx = 5x^2+x+6$.

For x^2, $A-2B = 5$ — (i)

x $3B-2C = 1$ — (ii)

x^0 $4A+3C = 6$ — (iii).

(i) $\times 4$ and (iii)

$\cancel{4A} - 8B = 20$

$\pm \cancel{4A} \pm 3C = \pm 6$

$-8B - 3C = 14$ — (iv) ~~and ii~~

(iv) $\times 2$ and (ii) $\times 3$.

$9B - \cancel{6C} = 3$

$\mp 16B \mp \cancel{6C} = -28$

$25B = -25$

$B = -1$.

$A = 5+2(-1) = 3$

$C = \dfrac{6-4(3)}{3} = -2$.

$\dfrac{3}{3-2x} - \dfrac{(x+2)}{x^2+4}$ (Ans)

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(11) $\frac{3}{3}\left(1 - \frac{2x}{3}\right)^{-1} - \frac{(x+2)}{4}\left(\frac{x^2}{4}+1\right)^{-1}$

$= 1 + \frac{2}{3}x + \frac{4}{9}x^2 - \frac{(x+2)}{4}\left(1 - \frac{x^2}{4}\right)$

$1 + \frac{2}{3}x + \frac{4}{9}x^2 - \frac{x}{4} + \frac{x^3}{16} - \frac{1}{2} + \frac{x^2}{8}$

not needed

$\frac{1}{2} + \frac{11}{12}x + \frac{41}{72}x^2$ (Ans).

~~$1 + \frac{2}{3}x + \frac{4}{9}x^2 - \frac{1}{4}\left(x - \frac{x^3}{4} + 2 - x^2/2\right)$~~

9 i) $\frac{dx}{dt} = \frac{xe^{-t}}{k+e^{-t}}$.

$\frac{1}{x}dx = \int \frac{e^{-t}}{k+e^{-t}}\,dt$

$\ln x = -\ln(k+e^{-t}) + C$

when $x = 10$, $t = 0$.

$\ln 10 = -\ln(k+1) + C$

$\ln 10(k+1) = C$.

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$\ln x = \ln 10(k+1) - \ln(k+e^{-t})$ (Ans).

(ii) when $x = 20$, $t = 1$

$$\ln 20 = \ln \frac{10(k+1)}{k+e^{-1}}$$

$$20 = 10\,\frac{k+1}{k+e}$$

$$2k + 2e = k + 1$$

$k = 1 - 2/e$, shown.

(iii) $\cancel{\frac{xe^{-t}}{k+e^{-t}} = 0}$ $x = e^{\ln \frac{10(k+1)}{k+e^{-t}}}$ $\cancel{-\ln(k+e^{-t})}$.

When organisms is max, $\frac{xe^{-t}}{k+e^{-t}}$ is max and $k+e^{-t}$ is min.

as with increasing t, both of these values increase, t can never reach 48.

10 OA $\begin{pmatrix}2\\-1\\3\end{pmatrix}$ OB $\begin{pmatrix}1\\1\\5\end{pmatrix}$. $r = \begin{pmatrix}1\\1\\2\end{pmatrix} + u\begin{pmatrix}3\\1\\-1\end{pmatrix}$

$AB = \begin{pmatrix}2\\-1\\3\end{pmatrix} - \begin{pmatrix}1\\1\\5\end{pmatrix}$ eq of AB $= \begin{pmatrix}1\\1\\5\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\-2\end{pmatrix}$

$= \begin{pmatrix}1\\-2\\-2\end{pmatrix}$

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if intersecting,

$$\begin{pmatrix} 1+3\mu \\ 1+\mu \\ 2-\mu \end{pmatrix} = \cancel{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}} \begin{pmatrix} 1+\lambda \\ 1-2\lambda \\ 5-2\lambda \end{pmatrix}$$

~~But as~~ $\cancel{1+3\mu = 1}$

$\cancel{3\mu} =$

$1+3\mu = 1+\lambda$. $\qquad 1+\mu = 1-2\lambda$

$3\mu-\lambda = 0 \quad -(1)$ $\qquad \mu+2\lambda = 0 \quad -(11) \times 3$

equate,

$$\begin{aligned} 3\cancel{\mu} - \lambda &= 0 \\ \pm \cancel{3\mu} \pm 6\lambda &= 0 \\ \hline \cancel{-7\lambda} &= 0 \\ \lambda &= 0 \end{aligned}$$

$\therefore \mu = 0$.

~~putti~~ $2-\mu = 5-2\lambda$ $\qquad$ putting values in,

$2-0 = 5-0$

$2 \neq 5$.

they do not intersect.

(ii) ~~a~~ Plane containing l.

$$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & -1 \end{pmatrix}$$

$= (-1-2)i - (-1\cancel{0}-6)j + (1-3)k$

$= -3i + 7j \cancel{0} - 2k$.

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$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 7 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 7 \\ -2 \end{pmatrix}$$

$7y - 3x - 2z = 0$ (Ans)