# MATHEMATICS

**Paper 9709/12**

**Paper 12**

## Key Message

Many candidates showed their working out which meant that wrong final answers could still receive credit for correct working. All candidates would benefit from following the advice to show all necessary working.

## General Comments

Many good and excellent scripts were seen but **Questions 2, 6** and **10(ii)** did prove to be a difficult challenge for many. The standard of presentation was generally good with candidates setting their work out in a clear readable fashion with very few candidates dividing the page into two vertically. Candidates found a number of questions to be reasonably straightforward but some questions proved more of a challenge. Most candidates appeared to have sufficient time to complete the paper although some good candidates failed to complete **Question 11** and were clearly working on it when time was called.

Candidates should be encouraged to look at the number of marks available for different parts of questions. A number of candidates spent a great deal of time unsuccessfully trying to obtain the one mark available in **Questions 5(i)** or **9(iii)**. It is also worth pointing out that centres and candidates should be aware that generally a question with parts labelled **(i),(ii)** and **(iii)** implies that there is a link between the parts as opposed to questions with parts labelled **(a)**, **(b)** and **(c)** where the link may not exist. In **Question 5** a number of candidates failed to use the result shown in **5(i)** in **5(ii)**.

## Comments on Specific Questions

### Question 1

This question was generally well answered by most candidates. Some candidates omitted the constant of integration or substituted for $x$ only. A minority of the candidates misunderstood the question and found the equation of a straight line through (3, 5), others misunderstood the notation and tried to find the inverse function by rearranging $y = 5 - 2x^2$ to make $x$ the subject. There were a number of minor errors even from good candidates. Candidates could perhaps be encouraged to go back and check question **1** again when they have finished the rest of the paper.

*Answer:* $\mathrm{f}(x) = 5x - \dfrac{2x^3}{3} + 8$

### Question 2

This question proved challenging for many candidates with a relatively small number of gaining the full 4 marks. Many demonstrated that they did not have a clear understanding of the structure of the diagram and what calculations were needed to find the required area. A reasonable number found the areas of the triangle and sector correctly but only stronger candidates were able to find the area of the semicircle. Recognising that the radius was $r\sin\theta$ proved problematic. A significant number used the cosine rule and a smaller number the sine rule to correctly find the length of $AB$ but then struggled to follow through square rooting, halving and substituting correctly to find the area. Some candidates incorrectly simplified $\frac{1}{2}r^2\sin 2\theta$ as $r^2\sin\theta$, or $\frac{1}{2}r^2 2\theta - \frac{1}{2}r^2\sin 2\theta$ as $2\theta - \sin 2\theta$. A significant number obviously spent a lot of time on the question with several attempts or replacement solutions.

*Answer:* $\frac{1}{2}\pi r^2\sin^2\theta - r^2\theta + \frac{1}{2}r^2\sin^2 2\theta$

**Question 3**

The majority of candidates gained full marks on this question. There were occasional errors with signs in part **(i)**, however many candidates were still able to get 2 marks in part **(ii)** due to the follow through available. In part **(ii)** it took some of the candidates a long time to reach the answer as they worked out the full expansion.

*Answer:* **(i)** 240, –160 ; **(ii)** 560.

**Question 4**

There was a mixed response to this question. Some candidates did not read the question carefully enough and tried to obtain $u$ as a function of $y$ instead of $x$. For those with good algebraic technique, it was straight-forward, but a good number failed to deal correctly with the denominator when substituting $y$ with $(12–x)/3$, resulting in an incorrect function of $x$. Some candidates got to $x = \mathrm{f}(u)$ but did not then differentiate but put $\mathrm{f}(u) = 0$. Many correct answers for the stationary value of $x$ were seen but fewer for the stationary value of $u$ as requested in the question.

*Answer:* 6

**Question 5**

Candidates were generally more successful with part **(ii)** than with part **(i)**. Candidates often made the question unnecessarily complicated by multiplying the numerator and denominator by either $\sin\theta + \cos\theta$ or by $\sin\theta - \cos\theta$ but then failed to cope with the resulting trigonometric algebra. More successful approaches were dividing the numerator and denominator on the left hand side by $\cos\theta$ or changing $\sin\theta$ to $\tan\theta\cos\theta$ and then factorising. Part **(ii)** was well answered with most candidates appreciating the link between part **(i)** and part **(ii)** of the question. A small number of candidates obtained the correct quadratic equation in $\tan\theta$ but then incorrectly factorised it to $(\tan\theta - 6)(\tan\theta + 1)$ resulting in incorrect angles. A small number of candidates had a fully correct method but then spoilt this by giving inaccurate answers or included extra incorrect answers.

*Answer:* **(ii)** 63.4, 71.6.

**Question 6**

This question proved to be the most difficult on the paper for many candidates. In part **(i)** a number did not consider the range of the cosine function and use $\cos kt = –1$ which would have given them the correct answer of 120m. Some candidates saw the word 'maximum' and differentiated. Others thought that the maximum height was 60m in spite of part **(iii)** asking when the height was above 90m. In part **(ii)** many candidates substituted $t = 30$ and $h = 0$ but were either unable to proceed further or found $\cos kt = 0$ and failed to consider the alternative answer of $2\pi$. A number showed a good appreciation of the situation by substituting $t = 15$ and $h = 120$. Part **(iii)** again proved challenging and only the most able candidates appreciated that there were two solutions required which then needed to be subtracted. A very small number of candidates correctly found the time from 90m to 120m and then doubled this.

Answer: **(i)** 120; **(iii)** 10.

**Question 7**

There was a mixed response to this question. Many candidates found it to be a very routine question and quickly obtained full marks. Some though seemed very confused by the geometrical situation and found the equations and intersections of irrelevant lines. Students would in many cases have benefitted from drawing a sketch.

In part **(i)**, many candidates calculated the gradient of AB correctly but then failed to use the gradient of the perpendicular together with the midpoint of AB for the required line. The points (4, 6) and (2, 10) were often observed being substituted into the equation of the line rather than the midpoint. The point (3,11) was often used with the perpendicular gradient rather than the parallel one by weaker candidates.

*Answer:* **(i)** $y - 4 = \frac{3}{2}(x - 7)$ **; (ii)** (9,7).

**Question 8**

Many candidates scored well on this question. In part **(i)** some candidates did not really know how to tackle the question systematically but the majority successfully found that d = – 3 and then used –22 = *a* + *(n–1) d* to find that *n* was 27 and then substituted this into the formula for the sum of *n* terms. Some candidates succeeded in equating the two expressions for $S_n$ with $d = -3$, i.e. $\frac{n}{2}(a + l) = \{2a + (n - 1)d\}$ but many candidates got bogged down with the algebra. Some thought that $d = 3$ instead of $-3$ and those who did use $d = -3$ sometimes failed to multiply out the bracket correctly, resulting in $n = 25$. Most were able to use either $S_n$ formula but some applied them with a negative or fractional value of *n* and received no credit for this.
In part **(ii)** most candidates who achieved full marks used the three terms to form expressions for *r* giving: $\frac{2k}{2k+6} = \frac{k+2}{2k}$ which led to the quadratic $2k^2 - 10k - 12 = 0$. Other candidates formed the more complex appearing expression $(2k+6)\frac{(2k)^2}{(2k+6)} = k+2$ and did not cancel down before expanding and often got into a mess with their algebra, or abandoned the cubic equation they had at the end. Some candidates successfully solved the cubic equation rejecting the inappropriate values for *k*. Other common errors were $(2k)^2$ becoming $2k^2$ or $(2k+6)^2$ becoming $4k^2+36$ .Some weak candidates "cancelled" $\frac{2k}{(2k+6)}$ to obtain $r = \frac{1}{6}$

In part **(iii)** nearly all candidates understood which equation to use for the sum to infinity but some tried to use a value of *r* which was greater than 1 and received no credit for this.

*Answer:* **(a)** 459; **(bi)** $k = 6$, **(ii)** 54.

**Question 9**

This was very well attempted by many candidates, particularly part **(i)**, but there were occasional errors in calculating the modulus or the scalar product.

In part **(ii)** most candidates were able to find **OC** successfully, but quite a few then stopped, and didn't go on to find a unit vector, achieving only 2 of the 4 marks.

There was often no attempt at part **(iii)**. As the lengths of **OA** and **OC** had been found earlier in the question little work was required. Some candidates, though, confused 'isosceles' for 'equilateral 'and others wasted a lot of time attempting, usually unsuccessfully, to show that the angles rather than the sides were equal.

*Answer:* **(i)** 31.8°; **(ii)** $\frac{1}{6}(4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$

**Question 10**

As was indicated in the key message above, the phrase: ‘showing all necessary working’ was deliberately inserted in part **(i)** so that candidates who simply obtained the volume of revolution by using the integration function on their calculators with no method shown, received no credit. Most candidates realised this and attempted to show their working although some simply wrote down the answer from their calculator and showed either no working or completely incorrect working. All but the weakest recognised that they needed to use $\pi y^2$ to integrate to find the volume and it was very pleasing to see many candidates realising the need to divide by 2 when integrating. Some weak students expanded the brackets and attempted to integrate the terms individually. The substitution of limits was general good, although the negative sign caused some arithmetical issues.

In part **(ii)** many candidates recognised that they should attempt to differentiate, but most did not realise that they should use the given line to identify the gradient of the normal, and thus find the gradient of the tangent, which they needed to equate to their differential. Some who did sometimes struggled with the algebraic manipulations, and many failed to solve the quadratic so that both values of $x$ were calculated, and as a result only achieved one pair of values. Many candidates, even some stronger ones, did not differentiate but incorrectly tried to equate the curve with the normal and then used the discriminant. A small number used this approach correctly, but with the tangent and the curve.

*Answer:* **(i)** $16\frac{\pi}{3}$; **(ii)** $\frac{5}{2}$ or $-\frac{7}{2}$

**Question 11**

There was a mixed response to this question. Those with strong algebraic skills found it straightforward but weaker candidates failed to make much progress. In part **(i)** some candidates forgot to take p to the left hand side and others struggled with the resulting expansion. Some managed it correctly but then put $p = \frac{1}{2}$ . In part **(ii)** many candidates were easily able to complete the square but weaker candidates struggled to cope with the $2x^2$ . As was mentioned in the general comments, a question with parts labelled **(i),(ii)** and **(iii),** etc. implies that there is a link between the parts. Weaker candidates failed to see the link between part **(ii)** and the final 3 parts of the question. In part **(iii)** some candidates put the range equal to a single value and others simply substituted in 0 and 4 and did not see the implications of part **(ii)**. In part **(v)** a common error was to try to rearrange the quadratic without completing the square.

*Answer:* **(i)** *p<1/2;* **(ii)** $2(x-\frac{3}{2})^2$ **+** $\frac{1}{2}$, **(iii)** $\frac{1}{2} \le \text{g}(x) \le 13$ **(iv)** $\frac{3}{2}$, (v) $\frac{3}{2}+\sqrt{\left(\frac{2x-1}{4}\right)}$