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Activity 11

Scheme of work example 9709

Learning objectives	Suggested teaching activities
Understand the meaning of $\|x\|$ and use relations such as $\|a\| = \|b\| \Leftrightarrow a^2 = b^2$ and $\|x - a\| < b \Leftrightarrow a - b < x < a + b$ in the course of solving equations and inequalities.	To introduce the notation, start with a numerical value, e.g. –5, and discuss the meaning of $\|-5\|$. You could help learners to deduce the results $\|a\| = \|b\| \Leftrightarrow a^2 = b^2$ and $\|x - a\| < b \Leftrightarrow a - b < x < a + b$ as part of a class discussion. This link leads to four files which are extremely useful. Click on https://www.tes.co.uk/teaching-resource/a-level-maths-c2-modulus-function-worksheets-6146818 and log in for free download. 'Modulus Function Introduction' provides a worksheet for learners to complete. **(I)** 'Solving Modulus Equations and Inequalities' could be used for consolidation/practice. **(I)** 'Modulus Transformations' provides practice at sketching graphs involving a modulus. You could demonstrate some initially to learners using a graph plotter. **(I)** 'Alternative Methods for Solving Modulus Equations' is a worksheet which helps learners to explore the different ways of solving this type of equation. **(I)** The link below demonstrates the graphs of various modulus functions. http://www.mathsmutt.co.uk/files/mod.htm **Past papers: (I)(F)** June 2014 paper 32, question 1 June 2014 paper 32, question 1 November 2014 paper 33, question 1 June 2013 paper 31, question 4 (involves logarithms) June 2013 paper 32, question 1
Divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero).	There are several different methods of polynomial division including inspection, the table method, and long division. This PowerPoint presentation introduces all three methods for factorising cubics. You can use the methods for any polynomial and also for division that results in a remainder: http://www.furthermaths.org.uk/files/sample/files/edx/Factorising_cubics.ppt When teaching any of the methods, start with a numerical example to remind learners of the thought process they

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	need, and use this to introduce the terms 'quotient' and 'remainder' . For example $54763 \div 8$ leads to a quotient of 6845 and a remainder of 3. Continue with a simple algebraic example $(x^2+4x+1)\div(x+2)$ which leads to a quotient of $x+2$ and a remainder of -3 . You will probably need to show learners further examples involving more complex polynomials before they practise on their own. The links below provide ideas on possible approaches you can take for long division: https://www.khanacademy.org/math/algebra2/polynomial_and_rational/dividing_polynomials/v/dividing-polynomials-with-remainders https://www.mathsisfun.com/algebra/polynomials-division-long.html This link has a work sheet of examples for practising any of the methods for division. http://www.mathworksheetsgo.com/sheets/algebra-2/polynomials/dividing-polynomials-worksheet.php **(I)** There is another approach known as synthetic division but learners have to be careful when using it, especially when factorising. You will find many useful questions in textbooks for learners to practise.
Use the factor theorem and the remainder theorem, e.g. to find factors, solve polynomial equations or evaluate unknown coefficients.	Summarise the work already done on polynomial division to show that $p(x)$ = (divisor $\times$ quotient) + remainder. Show that algebraic division can often be avoided in questions by substituting into $p(x)$ the value of x that makes the divisor zero (e.g. substituting 3 if the divisor is $x-3$ and calculating p(3) to find the remainder). Show that the factor theorem is a special case of the remainder theorem when the remainder is zero. The link below gives a good approach of this type which you could use with a whole class. https://www.mathsisfun.com/algebra/polynomials-remainder-factor.html You could show examples involving finding factors, solving polynomial equations and evaluating unknown coefficients to the whole class, questioning learners individually throughout. Remind learners that they should show all their working as the use of a calculator for finding solutions to polynomial equations will not be accepted in an exam. Here is a useful worksheet which covers basic use of the remainder theorem and evaluating unknown coefficients (log in for free download): https://www.tes.co.uk/teaching-resource/worksheet-on-the-remainder-theorem-6140286 **(I)** This link gives more examples on the remainder theorem and on solving polynomial equations. http://www.mash.dept.shef.ac.uk/Resources/A26remainder.pdf

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	Past papers: (I)(F) June 2014 paper 32, question 5 November 2014 paper 31, question 3 November 2014 paper 33, question 3 June 2013 paper 31, question 1 June 2013 paper 32, question 4
Recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than: - $(ax + b)(cx + d)(ex + f)$ - $(ax + b)(cx + d)^2$ - $(ax + b)(x^2 + c^2)$ and where the degree of the numerator does not exceed that of the denominator.	Examples of the three main types of partial fraction are here (log in for free download): https://www.tes.com/teaching-resource/partial-fractions-examples-6140352 This link shows some worked examples and there are 10 practice questions for learners to try at the end of the document. http://www.mathsisfun.com/algebra/partial-fractions.html Textbooks will also contain many examples for learners to practise. In many questions, the first part will involve breaking down rational functions into partial fractions and later parts will use partial fractions with another mathematical technique such as binomial expansion, integration or solving differential equations. You can set learners questions involving these topics when they have covered them. **Past papers: (I)(F)** June 2014 paper 31, question 9 (includes a binomial expansion) June 2014 paper 33, question 8 (includes integration) November 2014 paper 31, question 9 (includes a binomial expansion) November 2014 paper 32, question 9 (includes a binomial expansion) June 2013 paper 31, question 3 June 2013 paper 32, question 8 (includes differential equations).
Use the expansion of $(1 + x)^n$, where n is a rational number and $\|x\|<1$ (finding a general term is not included, but adapting the standard series to expand e.g. $\left(2-\frac{1}{2}x\right)^{-1}$ is included).	Learners have already met the binomial expansion in unit P1 so, to check their understanding, you could set them some preparatory questions on basic binomial expansions using the formula $(a+b)^n$, where n is a positive integer. **(I)** Ask learners to work out the first few terms of the expansion of $(1+x)^n$ from the formula for expanding $(a+b)^n$, to obtain $(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3$... This is now in a useful form for introducing negative

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	and fractional powers. The tutorial at this link shows that you need the condition $\lvert x\rvert < 1$ for negative powers because they generate an infinite series. The first few terms are only a good approximation if the values of *x* meet this condition and the series converges. http://www.examsolutions.net/maths-revision/core-maths/sequences-series/binomial/formula/validity/tutorial-1.php This link uses an example with *n* = $\frac{1}{2}$ and has an interesting graphical display of the approximation. http://www.intmath.com/series-binomial-theorem/4-binomial-theorem.php Textbooks will include many examples for learners to practise expanding and finding the range of values for which each expansion is valid.**(I)** You can demonstrate to learners how to re-write examples of the type $\left(2-\frac{1}{2}x\right)^{-1}$ as $\frac{1}{2}\left(1-\frac{x}{4}\right)^{-1}$ so that they can go on to expand them. **Past papers: (I)(F)** June 2014 paper 31, question 9 (includes partial fractions) June 2014 paper 33, question 2 November 2014 paper 31, question 9 (includes partial fractions) June 2013 paper 31, question 2