UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/02**

Paper 2 **October/November 2008**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

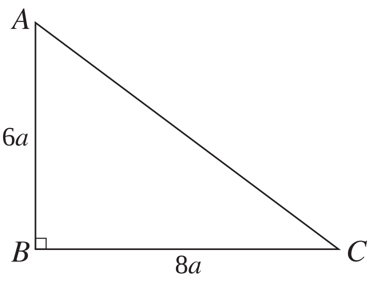
Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

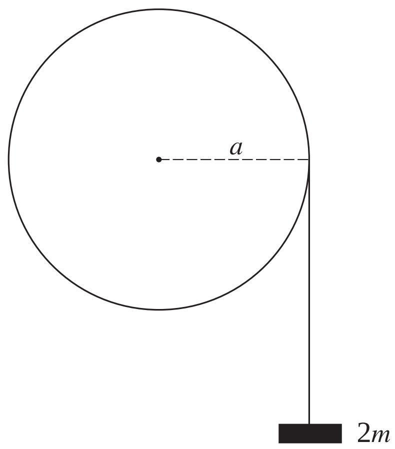
1



A uniform wire, of length $24a$ and mass m, is bent into the form of a triangle ABC with angle $ABC = 90°$, $AB = 6a$ and $BC = 8a$ (see diagram). Find the moment of inertia of the wire about an axis through A perpendicular to the plane of the wire. [5]

2 A small bead B of mass m is threaded on a smooth wire fixed in a vertical plane. The wire forms a circle of radius a and centre O. The highest point of the circle is A. The bead is slightly displaced from rest at A. When angle $AOB = \theta$, where $\theta < \cos^{-1}\left(\frac{2}{3}\right)$, the force exerted on the bead by the wire has magnitude R_1. When angle $AOB = \pi + \theta$, the force exerted on the bead by the wire has magnitude R_2. Show that $R_2 - R_1 = 4mg$. [8]

3



A uniform disc, of mass m and radius a, is free to rotate without resistance in a vertical plane about a horizontal axis through its centre. A light inextensible string has one end fixed to the rim of the disc, and is wrapped round the rim. A block of mass $2m$ is attached to the other end of the string (see diagram). The system is released from rest with the block hanging vertically. While the block moves it experiences a constant resistance to motion of magnitude $\frac{1}{10}mg$. Find the angular acceleration of the disc, and find also the angular speed of the disc when it has turned through one complete revolution. [9]

4 Two smooth spheres A and B, of equal radii, have masses 0.1 kg and m kg respectively. They are moving towards each other in a straight line on a smooth horizontal table and collide directly. Immediately before collision the speed of A is $5\,\mathrm{m\,s^{-1}}$ and the speed of B is $2\,\mathrm{m\,s^{-1}}$.

(i) Assume that in the collision A does not change direction. The speeds of A and B after the collision are $v_A\,\mathrm{m\,s^{-1}}$ and $v_B\,\mathrm{m\,s^{-1}}$ respectively. Express m in terms of v_A and v_B, and hence show that $m < 0.25$. [4]

(ii) Assume instead that $m = 0.2$ and that the coefficient of restitution between the spheres is $\frac{1}{2}$. Find the magnitude of the impulse acting on A in the collision. [6]

5 A particle of mass m moves in a straight line AB of length $2a$. When the particle is at a general point P there are two forces acting, one in the direction $\overrightarrow{PA}$ with magnitude $mg\left(\dfrac{PA}{a}\right)^{-\frac{1}{4}}$ and the other in the direction $\overrightarrow{PB}$ with magnitude $mg\left(\dfrac{PB}{a}\right)^{\frac{1}{2}}$. At time $t = 0$ the particle is released from rest at the point C, where $AC = 1.04a$. At time t the distance AP is $a + x$. Show that the particle moves in approximate simple harmonic motion. [4]

Using the approximate simple harmonic motion, find the speed of P when it first reaches the mid-point of AB and the time taken for P to first reach half of this speed. [7]

6 The independent random variables X and Y have normal distributions with the same variance σ^2. Samples of 5 observations of X and 10 observations of Y are made, and the results are summarised by $\Sigma x = 15$, $\Sigma x^2 = 128$, $\Sigma y = 36$ and $\Sigma y^2 = 980$. Find a pooled estimate of σ^2. [3]

7 The pulse rate of each member of a random sample of 25 adult UK males who exercise for a given period each week is measured in beats per minute. A 98% confidence interval for the mean pulse rate, μ beats per minute, for all such UK males was calculated as $61.21 < \mu < 64.39$, based on a t-distribution.

(i) Calculate the sample mean pulse rate and the standard deviation used in the calculation. [5]

(ii) State an assumption necessary for the validity of the confidence interval. [1]

(iii) The mean pulse rate for all UK males is 72 beats per minute. State, giving a reason, if it can be concluded that, on average, UK males who exercise have a reduced pulse rate. [2]

8 The equations of the regression lines for a random sample of 25 pairs of data (x, y) from a bivariate population are

$$y \text{ on } x: \quad y = 1.28 - 0.425x,$$
$$x \text{ on } y: \quad x = 1.05 - 0.516y.$$

(i) Find the sample means, $\bar{x}$ and $\bar{y}$. [3]

(ii) Find the product moment correlation coefficient for the sample. [2]

(iii) Test at the 5% significance level whether the population correlation coefficient differs from zero. [4]

9 A sample of 100 observations of the continuous random variable T was obtained and the values are summarised in the following table.

Interval	$1 \leqslant t < 1.5$	$1.5 \leqslant t < 2$	$2 \leqslant t < 2.5$	$2.5 \leqslant t < 3$
Frequency	64	17	16	3

It is required to test the goodness of fit of the distribution with probability density function given by

$$f(t) = \begin{cases} \dfrac{9}{4t^3} & 1 \leqslant t < 3, \\ 0 & \text{otherwise.} \end{cases}$$

The relevant expected values are as follows.

Interval	$1 \leqslant t < 1.5$	$1.5 \leqslant t < 2$	$2 \leqslant t < 2.5$	$2.5 \leqslant t < 3$
Expected frequency	62.5	21.875	10.125	5.5

Show how the expected value 10.125 is obtained. [3]

Carry out the test, at the 10% significance level. [7]

10 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} 0 & x < 0, \\ \dfrac{a}{2^x} & x \geqslant 0, \end{cases}$$

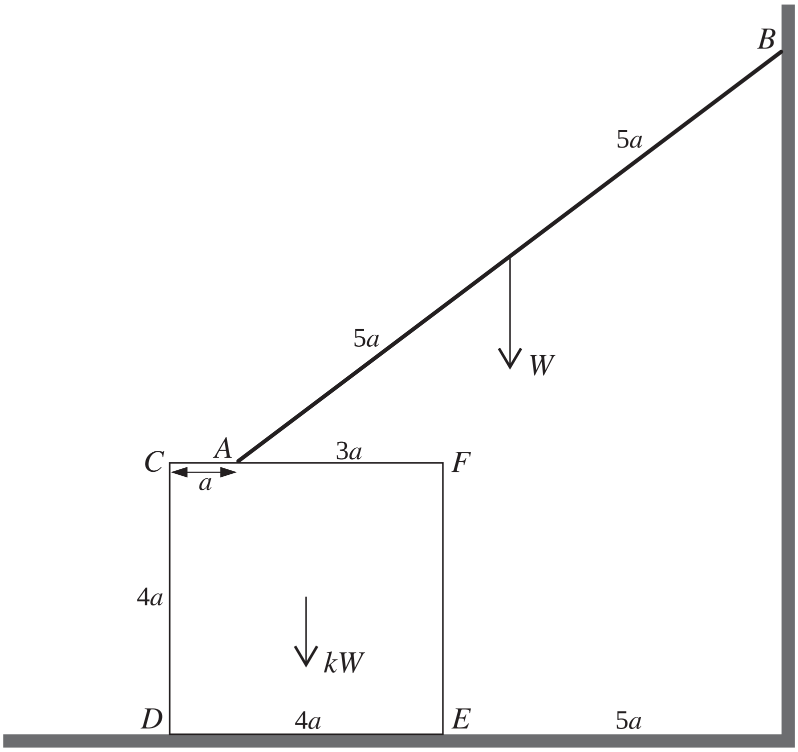
where a is a positive constant. By expressing 2^x in the form e^{kx}, where k is a constant, show that X has a negative exponential distribution, and find the value of a. [4]

State the value of $E(X)$. [1]

The variable Y is related to X by $Y = 2^X$. Find the distribution function of Y and hence find its probability density function. [8]

11 Answer only **one** of the following two alternatives.

EITHER



The diagram shows a central cross-section $CDEF$ of a uniform solid cube of weight kW with edges of length $4a$. The cube rests on a rough horizontal floor. One of the vertical faces of the cube is parallel to a smooth vertical wall and at a distance $5a$ from it. A uniform ladder, of length $10a$ and weight W, is represented by AB. The ladder rests in equilibrium with A in contact with the rough top surface of the cube and B in contact with the wall. The distance AC is a and the vertical plane containing AB is perpendicular to the wall. The coefficients of friction between the ladder and the cube, and between the cube and the floor, are both equal to μ. A small dog of weight $\frac{1}{4}W$ climbs the ladder and reaches the top without the ladder sliding or the cube turning about the edge through D. Show that $\mu \geqslant \frac{4}{5}$. [7]

Show that the cube does not slide whatever the value of k. [4]

Find the smallest possible value of k for which equilibrium is not broken. [3]

OR

A perfume manufacturer had one bottle-filling machine, but because of increased sales a second machine was obtained. In order to compare the performance of the two machines, a random sample of 50 bottles filled by the first machine and a random sample of 60 bottles filled by the second machine were checked. The volumes of the contents from the first machine, x_1 ml, and from the second machine, x_2 ml, are summarised by

$$\Sigma x_1 = 1492.0, \quad \Sigma x_1^2 = 44\,529.52, \quad \Sigma x_2 = 1803.6, \quad \Sigma x_2^2 = 54\,220.84.$$

The volumes have distributions with means μ_1 ml and μ_2 ml for the first and second machines respectively. Test, at the 2% significance level, whether μ_2 is greater than μ_1. [10]

Find the set of values of α for which there would be evidence at the α% significance level that $\mu_2 - \mu_1 > 0.1$. [4]

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