UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

# MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

## 9231 FURTHER MATHEMATICS

**9231/11** Paper 1, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

| Qu No | Commentary | Solution | Marks | Part Mark | Total |
|---|---|---|---|---|---|
| 1 | (N.B. Not $\alpha, \beta, \gamma$) | Let roots be $\alpha$, $\alpha$, and $\beta$. | | | |
| | Writes down sum of roots, | (1) $2\alpha + \beta = 0$ | M1 | | |
| | sum of products in pairs | (2) $2\alpha\beta + \alpha^2 = p$ | A1 | | |
| | and product of roots. | (3) $\alpha^2\beta = -q$ | A1 | | |
| | | From (1) $\beta = -2\alpha$ | | | |
| | Eliminates $\beta$ ( or $\alpha$ ). | (2) $\Rightarrow -4\alpha^2 + \alpha^2 = p \Rightarrow p = -3\alpha^2$ | | | |
| | | (3) $\Rightarrow -2\alpha^3 = -q \Rightarrow q = 2\alpha^3$ | M1 | | |
| | Equates power of $\alpha$ ( or $\beta$ ) | $\alpha^6 = \left(-\frac{p}{3}\right)^3 = \left(\frac{q}{2}\right)^2 \Rightarrow 4p^3 + 27q^2 = 0$ (AG) | A1 | 5 | [5] |
| 2 | Finds **a** × **b** | $\begin{vmatrix} i & j & k \\ 3 & 2 & -1 \\ 4 & -3 & 2 \end{vmatrix} = \mathbf{i} - 10\mathbf{j} - 17\mathbf{k}$ | M1 A1 | | |
| | Finds area of base. | $\frac{1}{2}\sqrt{1^2 + (-10)^2 + (-17)^2} = \frac{1}{2}\sqrt{390}$ ( = 9.87) | A1 | 3 | |
| | Attempts to find height | Height = $\frac{(3i - j - k).(i - 10j - 17k)}{\sqrt{1^2 + (-10)^2 + (-17)^2}} = \frac{30}{\sqrt{390}}$ ( = 1.519) | M1 | | |
| | Finds volume | $\frac{1}{3} \times \frac{1}{2}\sqrt{390} \times \frac{30}{\sqrt{390}} = 5$ | A1 | 2 | [5] |

| Qu No | Commentary | Solution | Marks | Part Mark | Total |
|---|---|---|---|---|---|
| 3 | Proves base case. | $H_n$ : $\dfrac{d^n}{dx^n}(e^x \sin x) = 2^{\frac{n}{2}} e^x \sin\left(x + \dfrac{n\pi}{4}\right)$ | | | |
| | | $\dfrac{d}{dx}(e^x \sin x) = \sin x e^x + e^x \cos x$ | M1 | | |
| | | $= \sqrt{2}e^x\left(\dfrac{\sin x}{\sqrt{2}} + \dfrac{\cos x}{\sqrt{2}}\right) = 2^{\frac{1}{2}} e^x \sin\left(x + \dfrac{\pi}{4}\right)$ | | | |
| | | $\Rightarrow H_1$ is true. | A1 | | |
| | States inductive hypothesis. | Assume $H_k$ is true : | B1 | | |
| | Proves inductive step. | $\dfrac{d^{k+1}}{dx^{k+1}}(e^x \sin x) = 2^{\frac{k}{2}}\left\{e^x \sin\left(x + \dfrac{k\pi}{4}\right) + e^x \cos\left(x + \dfrac{k\pi}{4}\right)\right\}$ | M1 | | |
| | | $= 2^{\frac{k+1}{2}} e^x \left\{\dfrac{1}{\sqrt{2}}\sin\left(x + \dfrac{k\pi}{4}\right) + \dfrac{1}{\sqrt{2}}\cos\left(x + \dfrac{k\pi}{4}\right)\right\}$ | A1 | | |
| | | $= 2^{\frac{k+1}{2}} e^x \left\{\sin\left(x + \dfrac{k\pi}{4} + \dfrac{\pi}{4}\right)\right\}$ | | | |
| | | $= 2^{\frac{k+1}{2}} e^x \sin\left(x + \dfrac{(k+1)\pi}{4}\right)$ | A1 | | |
| | States conclusion. | $\therefore H_k \Rightarrow H_{k+1}$<br>Hence true for all positive integers by PMI | A1 | 7 | [7] |

| Qu No | Commentary | Solution | Marks | Part Mark | Total |
|---|---|---|---|---|---|
| 4 (i) | Reduces matrix to echelon form. | $\begin{pmatrix} 3 & 4 & 2 & 5 \\ 6 & 7 & 5 & 8 \\ 9 & 9 & 9 & 9 \\ 15 & 16 & 14 & 17 \end{pmatrix} \sim \begin{pmatrix} 3 & 4 & 2 & 5 \\ 0 & -1 & 1 & -2 \\ 0 & -3 & 3 & -6 \\ 0 & -4 & 4 & -8 \end{pmatrix} \sim \begin{pmatrix} 3 & 4 & 2 & 5 \\ 0 & -1 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ | M1A1 | | |
| | States rank, | $\Rightarrow R(\mathbf{M}) = 2$. | A1 | | |
| | | Basis for range space is: | | | |
| | and basis for range space. | $\left\{ \begin{pmatrix} 3 \\ 6 \\ 9 \\ 15 \end{pmatrix}, \begin{pmatrix} 4 \\ 7 \\ 9 \\ 16 \end{pmatrix} \right\}$ (OE) | A1 | 4 | |
| | | **Alternatively:** | | | |
| | | $\mathbf{c}_1 = \begin{pmatrix} 3 \\ 6 \\ 9 \\ 15 \end{pmatrix}$ $\mathbf{c}_2 = \begin{pmatrix} 4 \\ 7 \\ 9 \\ 16 \end{pmatrix}$ $\mathbf{c}_3 = \begin{pmatrix} 2 \\ 5 \\ 9 \\ 14 \end{pmatrix}$ $\mathbf{c}_4 = \begin{pmatrix} 5 \\ 8 \\ 9 \\ 17 \end{pmatrix}$ | | | |
| | Shows linear dependence.<br>Finds a lin. indep. set.<br>States rank<br>and basis for range space. | $2\mathbf{c}_1 = \mathbf{c}_2 + \mathbf{c}_3$ and $\mathbf{c}_4 = \mathbf{c}_1 + \mathbf{c}_2 - \mathbf{c}_3 \Rightarrow$ lin. dep.<br>but any two e.g. $\mathbf{c}_1$, and $\mathbf{c}_2$ are lin. indep.<br>Hence $R(\mathbf{M}) = 2$<br>Basis of range space is $\{\mathbf{c}_1, \mathbf{c}_2\}$ | M1<br>A1<br>A1<br>A1 | | |
| (ii) | Forms equations. | $3x + 4y + 2z + 5t = 0$<br>$-y + z - 2t = 0$ | M1 | | |
| | (Gives two parameter solution.) | $(t = \lambda\ ,\ z = \mu\ ,\ y = \mu - 2\lambda\ ,\ x = \lambda - 2\mu)$ | | | |
| | States basis of null space. | Basis of null space is $\left\{ \begin{pmatrix} 1 \\ -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\}$ or $\left\{ \begin{pmatrix} -3 \\ 0 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -3 \\ 1 \\ 2 \end{pmatrix} \right\}$ | A1A1 | 3 | |
| | (Or by reducing transpose to echelon form, or by any other valid method.). | or any two of the above four vectors. | | | [7] |

| Qu No | Commentary | Solution | Marks | Part Mark | Total |
|---|---|---|---|---|---|
| 5 (i) | One mark for each side. | $3x^2 - 6y^2y' = 3y \quad + 3xy'$ | B1B1 | | |
| | Substitutes (2,1) | $12 - 6y' = 3 + 6y' \Rightarrow y' = \dfrac{3}{4}$ | B1√ | 3 | |
| (ii) | One mark for differentiating both 1st and 3rd terms. One mark for each of 2nd and 4th terms. | $6x - \{6y^2y'' + 12y(y')^2\} = 3y' + 3y' + 3xy''$ | B1B1 B1 | | |
| | Substitute (2,1) and $y'(2) = \dfrac{3}{4}$. | $12 - (6y'' + \dfrac{27}{4}) = \dfrac{9}{4} + \dfrac{9}{4} + 6y'' \Rightarrow 12y'' = \dfrac{3}{4} \Rightarrow y'' = \dfrac{1}{16}$ | B1 | 4 | [7] |
| 6 | | $I_n \ = \int_0^1 x^n(1-x)^{\frac{1}{2}}\,dx$ | | | |
| | Integrates by parts. | $= \left[-\dfrac{2}{3}x^n(1-x)^{\frac{3}{2}}\right]_0^1 + \dfrac{2}{3}\int_0^1 nx^{n-1}(1-x)(1-x)^{\frac{1}{2}}\,dx$ | M1A1 | | |
| | Rearranges. | $= 0 + \dfrac{2n}{3}\int_0^1 x^{n-1}(1-x)^{\frac{1}{2}}\,dx - \dfrac{2n}{3}\int_0^1 x^n(1-x)^{\frac{1}{2}}\,dx$ | M1A1 | | |
| | | $= \dfrac{2n}{3}I_{n-1} - \dfrac{2n}{3}I_n$ | | | |
| | Obtains printed result. | $\Rightarrow (2n+3)I_n \ = 2nI_{n-1}$ (AG) | A1 | 5 | |
| | Evaluates $I_0$. | $I_0 = \int_0^1 (1-x)^{\frac{1}{2}}\,dx = \left[-\dfrac{2}{3}(1-x)^{\frac{3}{2}}\right]_0^1 = \dfrac{2}{3}$ | B1 | | |
| | Uses reduction formula. | $I_3 = \dfrac{6}{9} \times \dfrac{4}{7} \times \dfrac{2}{5} \times \dfrac{2}{3} = \dfrac{32}{315}$ | M1A1 | 3 | [8] |

| Qu No | Commentary | Solution | Marks | Part Mark | Total |
|---|---|---|---|---|---|
| 7 | Vertical asymptote. | $x = 2$ | B1 | 3 | |
| | Divides by $(x - 2)$ | $y = x + p + 2 + \dfrac{2p+5}{x-2}$ | M1 | | |
| | Oblique asymptote. | $y = x + p + 2$ | A1 | 3 | |
| | Differentiates. | $\dfrac{dy}{dx} = \dfrac{x^2 - 4x + 4 - 2p - 5}{(x-2)^2}$ | M1A1 | | |
| | | $y' = 0 \Rightarrow x^2 - 4x - (2p + 1) = 0$ | M1 | | |
| | | $B^2 - 4AC > 0 \Rightarrow 16 + 4(2p + 1) > 0$ | M1 | | |
| | | $\Rightarrow p > -\dfrac{5}{2}$ | A1 | 5 | |
| | Sketches graph.<br>Working to show either $x^2 - x + 1 = 0$ has no real roots, or maximum value. | Axes and (0,–0.5) marked..<br>Upper Branch with minimum.<br>Lower with maximum below $x$-axis.<br>(Deduct at most 1 for poor forms at infinity.) | B1<br>B1<br>B1 | 3 | [11] |

| Qu No | Commentary | Solution | Marks | Part Mark | Total |
|---|---|---|---|---|---|
| 8 | Shows required result, using $\mathbf{Ae} = \lambda\mathbf{e}$. | $\mathbf{ABe} = \mathbf{A}\,\mu\mathbf{e} = \mu\mathbf{Ae} = \mu\lambda\mathbf{e} = \lambda\mu\mathbf{e}$ | M1A1 | 2 | |
| | States eigenvalues from leading diagonal. | Eigenvalues of **C** are –1, 1 and 2 | B1 | | |
| | Finds eigenvectors using cross-product or equations. M1A1 for first correct and A1 for the other two. | $\lambda = -1:\ \mathbf{e}_1 = \begin{vmatrix} i & j & k \\ 0 & -1 & 3 \\ 0 & 2 & 2 \end{vmatrix} = \begin{pmatrix} -8 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ | M1A1 | | |
| | | $\lambda = 1:\ \mathbf{e}_2 = \begin{vmatrix} i & j & k \\ -2 & -1 & 3 \\ 0 & 0 & 2 \end{vmatrix} = \begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ | | | |
| | | $\lambda = 2:\ \mathbf{e}_3 = \begin{vmatrix} i & j & k \\ -3 & 1 & 3 \\ 0 & -1 & 2 \end{vmatrix} = \begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix}$ | A1 | 4 | |
| | Uses $\mathbf{De} = \mu\mathbf{e}$. | $\mathbf{D}\begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ -12 \\ -6 \end{pmatrix} = -2\begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix}$ | M1A1 | | |
| | States eigenvalue. | Eigenvalue is –2. | A1 | 3 | |
| | Recognises that **CD** has an eigenvector common to **C** and **D** and | **CD** has an eigenvector $\begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix}$ | B1 | | |
| | states the corresponding eigenvalue. | and the corresponding eigenvalue is $-2 \times 2 = -4$. | B1√ | 2 | [11] |

| Qu No | Commentary | Solution | Marks | Part Mark | Total |
|---|---|---|---|---|---|
| 9 (i) | Uses mean value formula and integrates. | M.V. $=\dfrac{\int_0^{\ln 5}\frac{1}{2}\left(e^x+e^{-x}\right)dx}{\ln 5-0}=\dfrac{\left[\frac{1}{2}\left(e^x-e^{-x}\right)\right]_0^{\ln 5}}{\ln 5}$ | M1A1 | | |
| | Substitutes limits and evaluates. | $=\dfrac{\frac{1}{2}\left(5-\frac{1}{5}\right)}{\ln 5}=\dfrac{12}{5\ln 5}$ (= 1.49) | M1A1 | 4 | |
| (ii) | Differentiates and finds $1+(y')^2$. | $y'=\frac{1}{2}\left(e^x-e^{-x}\right)\Rightarrow 1+(y')^2=\left\{\frac{1}{2}\left(e^x+e^{-x}\right)\right\}^2$ | M1A1 | | |
| | Integrates and obtains result. | $s=\frac{1}{2}\int_0^{\ln 5}\left(e^x+e^{-x}\right)dx=\frac{1}{2}\left[e^x-e^{-x}\right]_0^{\ln 5}$ <br> $=\frac{1}{2}\left[5-\frac{1}{5}\right]=\frac{12}{5}$ | M1A1 | 4 | |
| (iii) | Uses surface area formula. | $S=2\pi\int_0^{\ln 5}\frac{1}{2}\left(e^x+e^{-x}\right)\cdot\frac{1}{2}\left(e^x+e^{-x}\right)dx$ | M1 | | |
| | Integrates. | $=\frac{\pi}{2}\int_0^{\ln 5}(e^{2x}+2+e^{-2x})dx$ <br> $=\frac{\pi}{2}\left[\frac{e^{2x}}{2}+2x-\frac{e^{-2x}}{2}\right]_0^{\ln 5}$ | M1 | | |
| | Substitutes limits and | $=\frac{\pi}{2}\left\{\left[\frac{25}{2}+2\ln 5-\frac{1}{50}\right]-\left[\frac{1}{2}+0-\frac{1}{2}\right]\right\}$ | A1 | | |
| | evaluates. | $=\pi\left(\frac{156}{25}+\ln 5\right)$ (=24.7) | A1 | 4 | [12] |

| Qu No | Commentary | Solution | Marks | Part Mark | Total |
|---|---|---|---|---|---|
| 10 | C: | Closed loop through (5,0) and $(1,\pi)$ | B1 | 3 | |
| | | Correct shape near $(1,\pi)$ | B1 | | |
| | Straight line | Perpendicular to initial line , through (2,0) | B1 | | |
| | | $\Rightarrow (3 + 2\cos\theta)\cos\theta = 2$ | | | |
| | Forms quadratic equation in usual form. | $\Rightarrow 2\cos^2\theta + 3\cos\theta - 2 = 0$ (aef) | M1 | | |
| | | $\Rightarrow (2\cos\theta - 1)(\cos\theta + 2) = 0$ | | | |
| | Solves quadratic equation. | $\Rightarrow \cos\theta = 0.5$ (since $\cos\theta > 0$) | A1 | | |
| | Writes down points of intersection. | Intersections at $\left(4, \frac{\pi}{3}\right)$ and $\left(4, -\frac{\pi}{3}\right)$. | A1A1 | 4 | |
| | | Calling points of intersection $A$ and $B$ and the pole $O$. Required area is two congruent sectors between $l$ and $C$ plus triangle $OAB$. | | | |
| | Finds required area. | Two sectors $= 2 \times \frac{1}{2}\int_{\frac{\pi}{3}}^{\pi} (9 + 12\cos\theta + 4\cos^2\theta)d\theta$ | M1 | | |
| | | $= \int_{\frac{\pi}{3}}^{\pi} (11 + 12\cos\theta + 2\cos 2\theta)d\theta$ | A1 | | |
| | | $= \left[11\theta + 12\sin\theta + \sin 2\theta\right]_{\frac{\pi}{3}}^{\pi}$ | M1 | | |
| | | $= \frac{22\pi}{3} - \frac{13\sqrt{3}}{2} = (11.78)$ | A1 | | |
| | | Triangle $= 2\sqrt{3} \times 2 = 4\sqrt{3} = (6.928)$ | B1 | | |
| | | Total Area $= \frac{22\pi}{3} - \frac{5\sqrt{3}}{2} = (18.708 = 18.7$ *(3sf)*$)$ | A1 | 6 | [13] |

| Qu No | Commentary | Solution | Marks | Part Mark | Total |
|---|---|---|---|---|---|
| 11 | **EITHER** | | | | |
| | Verifies that $\omega$ is a root. | $\left(\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}\right)^5+1=\cos\pi+i\sin\pi+1=0$ | B1 | | |
| | Factorises. | $(\omega^5+1)=(\omega+1)(\omega^4-\omega^3+\omega^2-\omega+1)=0$ | | | |
| | | $\omega\neq-1\Rightarrow\omega^4-\omega^3+\omega^2-\omega+1=0$ | | | |
| | | $\Rightarrow\omega^4-\omega^3+\omega^2-\omega=-1$ | B1 | 2 | |
| | | $\omega=\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}$ | | | |
| | Finds $\omega^4$ | $\Rightarrow\omega^4=\cos\frac{4\pi}{5}+i\sin\frac{4\pi}{5}=-\cos\frac{\pi}{5}+\sin\frac{\pi}{5}$ | M1 | | |
| | and subtracts. | $\Rightarrow\omega-\omega^4=2\cos\frac{\pi}{5}$ | A1 | | |
| | Finds $\omega^3$ | $\omega^3=\cos\frac{3\pi}{5}+i\sin\frac{3\pi}{5}$ | | | |
| | and $\omega^2$ | $\omega^2=\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}=\cos\frac{3\pi}{5}-i\sin\frac{3\pi}{5}$ | M1 | | |
| | and subtracts | $\omega^3-\omega^2=2\cos\frac{3\pi}{5}$ | A1 | 4 | |
| | | $-2\cos\frac{\pi}{5}-2\cos\frac{3\pi}{5}=-1\Rightarrow\cos\frac{\pi}{5}+\cos\frac{3\pi}{5}=\frac{1}{2}$ | M1A1 | | |
| | | $\cos\frac{\pi}{5}\cos\frac{3\pi}{5}=\frac{1}{4}(\omega-\omega^4)(\omega^3-\omega^2)$ | M1 | | |
| | | $=\frac{1}{4}(\omega^4-\omega^3-\omega^7+\omega^6)$ | | | |
| | | $=\frac{1}{4}(\omega^4-\omega^3+\omega^2-\omega)=-\frac{1}{4}$ | A1 | 4 | |
| | | Equation with roots $\cos\frac{\pi}{5}$ and $\cos\frac{3\pi}{5}$ is: | | | |
| | Finds required quadratic equation. | $x^2-\frac{1}{2}x-\frac{1}{4}=0$ or $4x^2-2x-1=0$ | M1 | | |
| | Solves for $x$. | $\Rightarrow x=\frac{2\pm2\sqrt{5}}{8}$ | M1A1 | | |
| | States required value. | $\Rightarrow\cos\frac{\pi}{5}=\frac{1+\sqrt{5}}{4}$ (since $0<\cos\frac{\pi}{5}<1$) | A1 | 4 | **[14]** |

| Qu No | Commentary | Solution | Marks | Part Mark | Total |
|---|---|---|---|---|---|
| 11 | **OR** | | | | |
| | Differentiates | $z = x^2 y \Rightarrow \frac{dz}{dx} = x^2\frac{dy}{dx} + 2xy$ | M1 | | |
| | twice. | $\Rightarrow \frac{d^2z}{dx^2} = x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y$ | A1 | | |
| | Rearranges LHS of DE. | $\therefore x^2\frac{d^2y}{dx^2} + 4x(1+x)\frac{dy}{dx} + (2+8x+4x^2)y =$ | M1 | | |
| | | $\left(x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y\right) + 4\left(x^2\frac{dy}{dx} + 2xy\right) + 4x^2y$ | | | |
| | | $= \frac{d^2z}{dx^2} + 4\frac{dz}{dx} + 4z = 8x^2$ (AG) | A1 | 4 | |
| | Finds and solves AQE. | $m^2 + 4m + 4 = 0 \Rightarrow (m+2)^2 = 0 \Rightarrow m = -2$ | M1 | | |
| | States CF | CF: $z = Ae^{-2x} + Bxe^{-2x}$ | A1 | | |
| | States form of PI. | PI: $z = ax^2 + bx + c$ | | | |
| | Differentiates twice, | $\Rightarrow z' = 2ax + b \Rightarrow z'' = 2a$ | M1 | | |
| | substitutes | $2a + 8ax + 4b + 4ax^2 + 4bx + 4c = 8x^2$ | A1 | | |
| | and equates coefficients. | $2a + 4b + 4c = 0$ | M1 | | |
| | | $8a + 4b = 0$ | | | |
| | | $4a = 8$ | | | |
| | Solves. | $a = 2 \quad b = -4 \quad c = 3$ | A1 | | |
| | States GS for *z*–*x*. | $z = Ae^{-2x} + Bxe^{-2x} + 2x^2 - 4x + 3$ | M1 | | |
| | States GS for *y*–x. | $y = \frac{A}{x^2}e^{-2x} + \frac{B}{x}e^{-2x} + 2 - \frac{4}{x} + \frac{3}{x^2}$ | A1 | 8 | |
| | Considers the effect of $x \to \infty$. | As $x \to \infty$, $e^{-2x}$, $\frac{1}{x}$ *and* $\frac{1}{x^2} \to 0$ | M1 | | |
| | | $\therefore y \to 2$ | A1 | 2 | **[14]** |