Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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*6191011311*

**FURTHER MATHEMATICS** **9231/22**

Paper 2 Further Pure Mathematics 2 **May/June 2021**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

**1** **(a)** Given that $a$ is an integer, show that the system of equations

$$\begin{aligned} ax + 3y + z &= 14, \\ 2x + y + 3z &= 0, \\ -x + 2y - 5z &= 17, \end{aligned}$$

has a unique solution and interpret this situation geometrically. [4]

**(b)** Find the value of $a$ for which $x = 1,\ y = 4,\ z = -2$ is the solution to the system of equations in part **(a)**. [1]

**2** The variables $x$ and $y$ are related by the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 3\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 2x + 1.$$

**(a)** Find the general solution for $y$ in terms of $x$. [6]

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**(b)** State an approximate solution for large positive values of $x$. [1]

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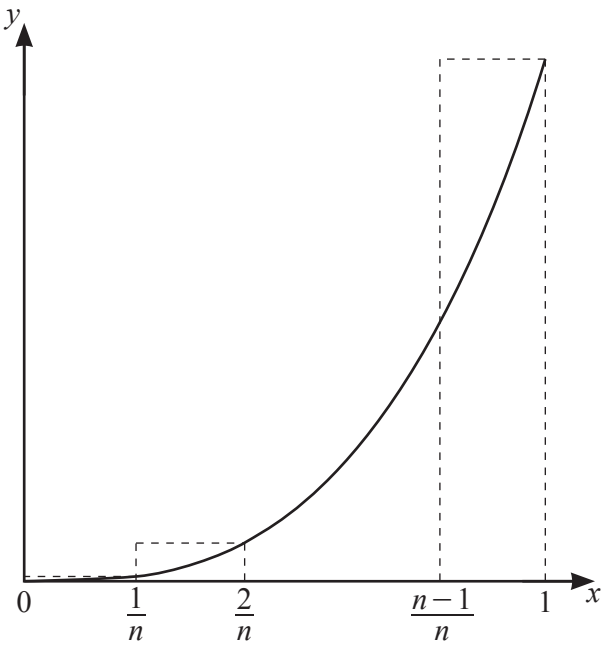
**3**




The diagram shows the curve with equation $y = x^3$ for $0 \leqslant x \leqslant 1$, together with a set of $n$ rectangles of width $\frac{1}{n}$.

**(a)** By considering the sum of the areas of these rectangles, show that $\int_0^1 x^3 \mathrm{d}x < U_n$, where

$$U_n = \left(\frac{n+1}{2n}\right)^2. \qquad [4]$$

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**(b)** Use a similar method to find, in terms of $n$, a lower bound $L_n$ for $\int_0^1 x^3 \mathrm{d}x$. [4]

**(c)** Find the least value of $n$ such that $U_n - L_n < 10^{-3}$. [2]

**4** Find the solution of the differential equation

$$\sin\theta\frac{\mathrm{d}y}{\mathrm{d}\theta}+y=\tan\tfrac{1}{2}\theta,$$

where $0<\theta<\pi$, given that $y=1$ when $\theta=\frac{1}{2}\pi$. Give your answer in the form $y=\mathrm{f}(\theta)$. [9]

[You may use without proof the result that $\int\operatorname{cosec}\theta\,\mathrm{d}\theta=\ln\tan\tfrac{1}{2}\theta$.]

[Turn over

**5** **(a)** State the sum of the series $z+z^2+z^3+...+z^n$, for $z \neq 1$. [1]

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**(b)** Given that $z$ is an $n$th root of unity and $z \neq 1$, deduce that $1+z+z^2+...+z^{n-1}=0$. [2]

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**(c)** Given instead that $z=\frac{1}{3}(\cos\theta+\mathrm{i}\sin\theta)$, use de Moivre's theorem to show that

$$\sum_{m=1}^{\infty} 3^{-m}\cos m\theta = \frac{3\cos\theta-1}{10-6\cos\theta}.$$ [7]

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**6** The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 5 & -\frac{22}{3} & 8 \\ 0 & -6 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

**(a)** Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^2 = \mathbf{PDP}^{-1}$. [7]

**(b)** Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^3$. [4]

**7** **(a)** It is given that $y = \text{sech}^{-1}\left(x+\frac{1}{2}\right)$.

Express cosh $y$ in terms of $x$ and hence show that $\sinh y \dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{\left(x+\frac{1}{2}\right)^2}$. [3]

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**(b)** Find the first three terms in the Maclaurin's series for $\text{sech}^{-1}\left(x+\frac{1}{2}\right)$ in the form

$$\ln a + bx + cx^2,$$

where $a$, $b$ and $c$ are constants to be determined. [7]

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**8** The curve $C$ has parametric equations

$$x = 2\cosh t, \quad y = \tfrac{3}{2}t - \tfrac{1}{4}\sinh 2t, \quad \text{for } 0 \leqslant t \leqslant 1.$$

**(a)** Find $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ and show that $\dfrac{\mathrm{d}y}{\mathrm{d}t} = 1 - \sinh^2 t$. [3]

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The area of the surface generated when $C$ is rotated through $2\pi$ radians about the $x$-axis is denoted by $A$.

**(b) (i)** Show that $A = \pi \int_0^1 \left(\tfrac{3}{2}t - \tfrac{1}{4}\sinh 2t\right)(1 + \cosh 2t)\,\mathrm{d}t$. [4]

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**(ii)** Hence find $A$ in terms of $\pi$, $\sinh 2$ and $\cosh 2$. [6]

## Additional Page

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