



# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.







3 The sequence of real numbers  $a_1, a_2, a_3, \dots$  is such that  $a_1 = 1$  and

$$a_{n+1} = \left(a_n + \frac{1}{a_n}\right)^3.$$

(a) Prove by mathematical induction that  $\ln a_n \geq 3^{n-1} \ln 2$  for all integers  $n \geq 2$ . [6]

[You may use the fact that  $\ln\left(x + \frac{1}{x}\right) > \ln x$  for  $x > 0$ .]

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(b) Show that  $\ln a_{n+1} - \ln a_n > 3^{n-1} \ln 4$  for  $n \geq 2$ . [2]

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5 The plane  $\Pi$  has equation  $\mathbf{r} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} + \mathbf{k}) + \mu(2\mathbf{i} + 3\mathbf{j})$ .

(a) Find a Cartesian equation of  $\Pi$ , giving your answer in the form  $ax + by + cz = d$ . [4]

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The line  $l$  passes through the point  $P$  with position vector  $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$  and is parallel to the vector  $\mathbf{k}$ .

(b) Find the position vector of the point where  $l$  meets  $\Pi$ . [3]

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(c) Find the acute angle between  $l$  and  $\Pi$ . [3]

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(d) Find the perpendicular distance from  $P$  to  $\Pi$ . [3]

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7 The curve  $C$  has equation  $y = \frac{4x+5}{4-4x^2}$ .

(a) Find the equations of the asymptotes of  $C$ . [2]

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(b) Find the coordinates of any stationary points on  $C$ . [4]

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(c) Sketch  $C$ , stating the coordinates of the intersections with the axes.

[3]

(d) Sketch the curve with equation  $y = \left| \frac{4x+5}{4-4x^2} \right|$  and find in exact form the set of values of  $x$  for which  $4|4x+5| > 5|4-4x^2|$ .

[6]

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