



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

THINKING SKILLS

9694/32

Paper 3 Problem Analysis and Solution

May/June 2013

1 hour 30 minutes

Additional Materials: Answer Booklet/Paper
 Electronic Calculator

* 2 0 0 9 5 5 8 1 7 7 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

Do not use staples, paper clips, highlighters, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.

Start each question on a new answer sheet.

Calculators should be used where appropriate.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

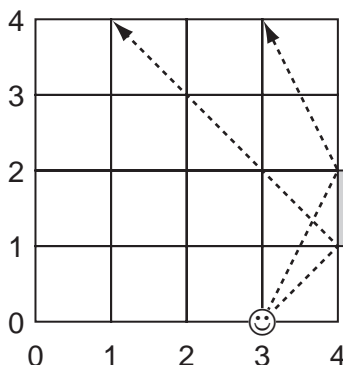
This document consists of **8** printed pages.



1 Study the information below and answer the questions. Show your working.

A retro computer game requires the player to destroy targets by shooting lasers. These can be reflected off mirrors on the walls in order to score more points. If the player hits the target directly he scores one point. The score increases by one point every time that the laser beam rebounds off a mirror on its way to the target. If the player is struck by his own laser beam then the game is over.

The player and the targets can only be positioned at integer coordinate points on the grid. The positions are referred to using the bottom edge and the left-hand edge as axes. For example, in the diagram below the player (\odot) is presented as shooting lasers from $(3, 0)$.



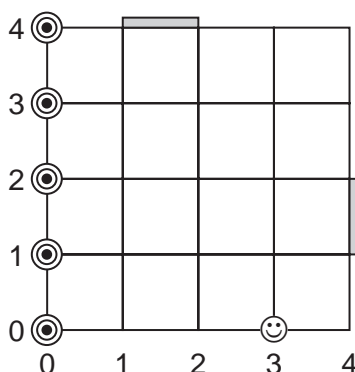
The player positioned at $(3, 0)$ is able to hit targets at $(1, 4)$, $(2, 4)$ and $(3, 4)$ on the top wall, using the mirror.

The first level of the game features a room with only one mirror, positioned as in the example above.

(a) Suggest two places where a player could be positioned in order to hit a target at $(0, 4)$ by rebounding off the mirror. [1]

(b) Which position allows a player to hit targets in the greatest number of possible positions, having rebounded the laser beam off the mirror? [1]

At level 2, an extra mirror is added on the top wall between $(1, 4)$ and $(2, 4)$. There is a target in each of the five positions along the left-hand wall of the room, and the player is positioned at $(3, 0)$.



(c) What is the greatest score that he can achieve? State how many points he scores for each target. [3]

At level 3 a third mirror is introduced on the bottom wall, between (3, 0) and (4, 0).

- (d) The player is positioned at (3, 3). At what positions along the top wall can he hit targets by rebounding off one or more mirrors? [2]

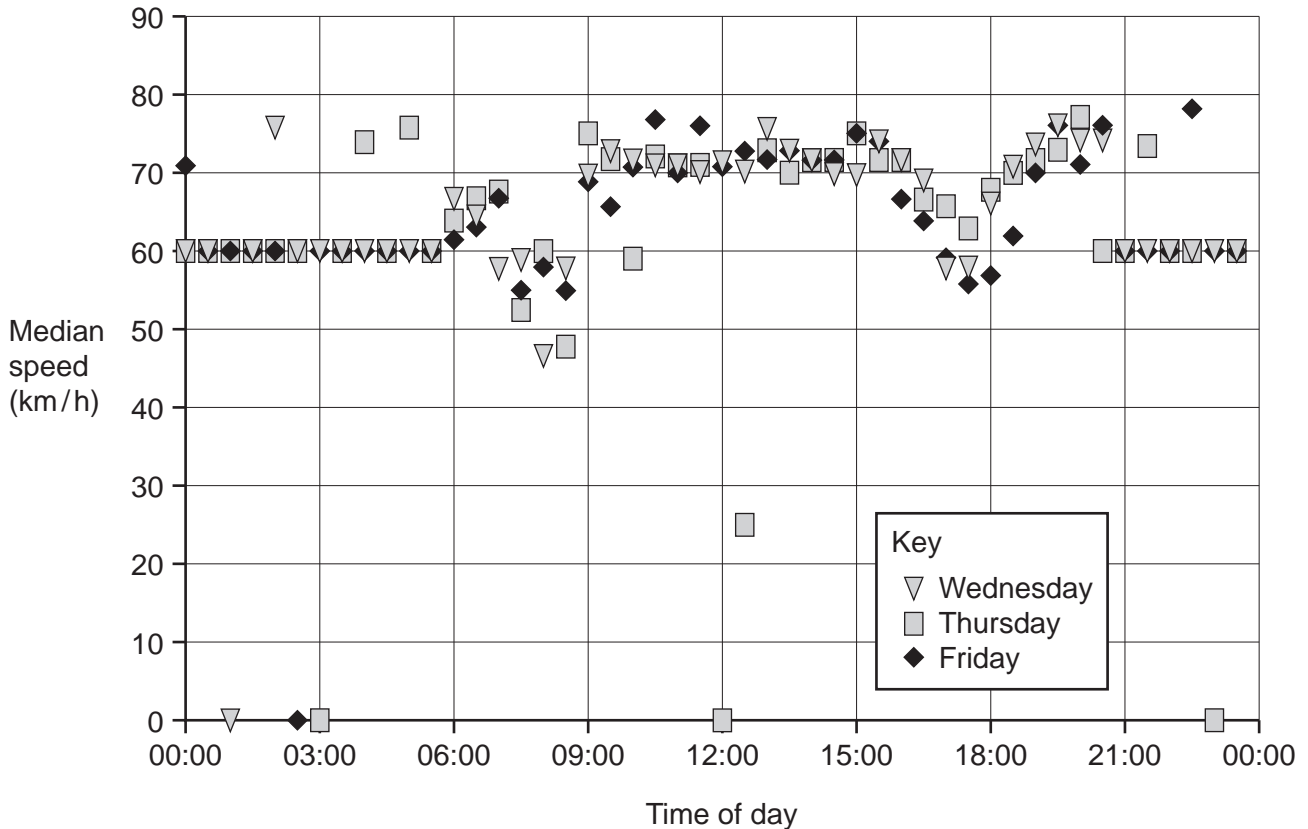
At level 4 no extra mirrors are added, but the player is now able to move horizontally or vertically. However, there is a penalty of one point per unit moved.

- (e) The player is originally positioned at (2, 1), and can destroy a target at (0, 1) by rebounding off the top mirror, scoring two points. Explain precisely how the player can destroy the target and score more points overall. [2]
- (f) In what position is a player capable of shooting himself using all three mirrors with just one shot? [1]

2 Study the information below and answer the questions. Show your working.

The speed of traffic passing a camera on a multi-lane highway was recorded for one minute every half hour for three consecutive days, and the **median** speeds plotted below.

The road has two distinct types of users: there are trucks, limited to travelling at 60 km/h (and travelling at exactly 60 km/h when traffic is flowing freely); and cars at speeds from 70 to at most 80 km/h, unless there is very heavy traffic or an accident.



- (a) The median speed on Friday at 22:30 was 78 km/h. During this minute, just one truck and two cars were recorded. Suggest speeds at which the three vehicles could have been travelling. [1]
- (b) Which one of the five points at 0 km/h is most plausibly due to an accident, rather than there being no traffic at all? Explain your answer. [2]
- (c) For some values of the median, one can be certain that the traffic was **not** flowing freely. What are these possible values for the median? [2]
- (d) Between 01:00 and 05:00 on the three days, how many cases (if any) could there be where just one truck and one car passed the camera during the minute? How can you tell? [2]
- (e) At 14:00 on each day the median is 72 km/h. What are the maximum and minimum possible values of the **mean** speed of the traffic at 14:00? [3]

3 Study the information below and answer the questions. Show your working.

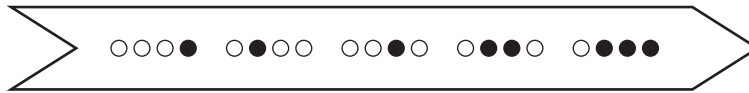
In the car park of Sydenham Station, the machine for producing parking tickets punched holes in a paper ticket to indicate the time at the end of the period that had been paid for, using the 24-hour clock. Each digit was encoded separately as shown:

0	○ ○ ○ ○	5	○ ● ○ ●
1	○ ○ ○ ●	6	○ ● ● ○
2	○ ○ ● ○	7	○ ● ● ●
3	○ ○ ● ●	8	● ○ ○ ○
4	○ ● ○ ○	9	● ○ ○ ●

● hole punched
○ no hole

An extra digit – a check-digit – was also placed at the end as a simple check against tampering. The check-digit was chosen so that the sum of all five digits was a multiple of 10. The shape of the ticket was designed so that the direction of reading was clear.

For example, a ticket valid until 14:26 would be given as 14267 and punched with 8 holes as shown:



This gave the opportunity for fraudulent tampering with the tickets. For example, the digit 1 could be changed to a 3 by punching another hole, and a 0 could be made into a 7 by punching 3 more holes.

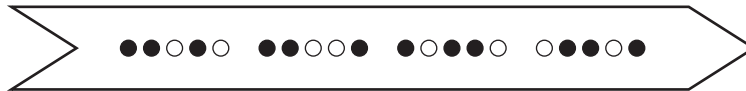
- (a) Which of the ten possible digits could never be changed to any other? [2]
- (b) (i) Give an example of a 5-digit ticket number where the digits in two positions can be altered to give another valid ticket (i.e. with a valid time and sum). Show both the original and amended ticket numbers. [3]
- (ii) What is the greatest time that can be fraudulently gained with only two extra holes punched? [2]
- (c) There are cases where a ticket with a time of the form $0w:0x$ can be amended to a valid ticket with a later time $0y:0z$, without changing the check-digit. How many such cases are there? [3]

Bill suggested that by using five positions for each digit, and no check-digit, it would be possible to use a coding which could not be changed to any other by punching extra holes. This was the specific suggestion:

0	●●●○○	5	●○○○●
1	●●○●○	6	○●●○●
2	●○●●○	7	●○○●●
3	○●●●○	8	○●○●●
4	●●○○●	9	○○●●●

● hole punched
○ no hole

14:26 would then be



Using the suggested system would produce more chads (the small circles of paper resulting from holes being punched), and so it would make more litter and the machines would wear out earlier.

- (d) (i) Suggest a variant of the five-position code which does not give any opportunity for fraud, but **always** produces fewer chads than the one suggested by Bill. [1]
- (ii) How many fewer chads than Bill's suggested system would your variant produce for a ticket valid to 14:26? [1]

There are 20 positions where a hole may be punched. It was pointed out that 18 positions would suffice for any time.

- (e) (i) Identify the two positions which would not be needed. [1]
- (ii) Swapping the codes for two particular digits would allow a further position to be removed. Which two codes could be swapped, and which position would no longer be needed? [2]

4 Study the information below and answer the questions. Show your working.

There are 9 teams in the Ophidian Football League. They all play each other twice, over an 18-week period, every season.

This table details the remaining fixtures of the current season, together with the results of all the matches played so far (home team's score first).

		Away team								
		Asps	Boomslangs	Cobras	Copperheads	Cottonmouths	Kraits	Mambas	Taipans	Vipers
Home team	Asps		Week 15	1 – 4	1 – 0	Week 12	Week 14	1 – 1	Week 18	1 – 2
	Boomslangs	0 – 1		1 – 3	Week 13	Week 14	0 – 2	Week 17	0 – 3	2 – 0
	Cobras	Week 17	Week 12		0 – 1	0 – 0	4 – 2	0 – 1	1 – 2	Week 14
	Copperheads	3 – 0	1 – 0	Week 15		Week 17	2 – 2	1 – 0	2 – 2	2 – 1
	Cottonmouths	2 – 1	2 – 3	Week 13	0 – 1		Week 16	1 – 4	1 – 3	Week 15
	Kraits	3 – 2	5 – 2	0 – 1	Week 18	0 – 0		2 – 1	Week 15	Week 17
	Mambas	Week 16	2 – 0	Week 18	3 – 2	4 – 0	Week 13		2 – 1	Week 12
	Taipans	2 – 2	Week 16	1 – 0	Week 12	2 – 0	3 – 3	Week 14		2 – 0
	Vipers	3 – 1	Week 18	2 – 2	Week 16	2 – 1	2 – 3	0 – 1	Week 13	

Teams are awarded 3 points for a win, and 1 point for a draw. In addition, 1 point is awarded to any team that scores 2 goals or more, regardless of the result of the match. For example, the Cobras' 4 – 2 defeat of the Kraits earned the Cobras a total of 4 points and the Kraits 1 point.

- (a) Which team will the Kraits play in their next
- (i) home match? [1]
- (ii) away match? [1]
- (b) Which is the only team so far this season to have
- (i) beaten the Taipans? [1]
- (ii) lost to the Cottonmouths? [1]
- (iii) scored at least one goal in every match? [1]
- (c) Because there is an odd number of teams in the league, one team each week has no fixture. This week (week 12) the Kraits are without a match, then next week (week 13) the Asps don't play.

For each of the last 5 weeks of the season (weeks 14 – 18), state which team has no fixture. [3]

This is the latest league table as published in today's Ophidian Gazette.

	<i>Played</i>	<i>Won</i>	<i>Drawn</i>	<i>Lost</i>	<i>Goals For</i>	<i>Goals Against</i>	<i>Points</i>
<i>Taipans</i>	10	6	3	1	21	11	29
<i>Mambas</i>	10	7	1	2	19	8	27
<i>Kraits</i>	10	5	3	2	22	17	26
<i>Copperheads</i>	10	6	2	2	15	9	25
<i>Cobras</i>	10	4	2	4	15	11	18
<i>Asps</i>	10	2	2	6	11	20	10
<i>Boomslangs</i>	9	2	0	7	8	19	9
<i>Cottonmouths</i>	10	1	2	7	7	20	7

- (d) The table is incomplete, as the line for the Vipers has been omitted.

Work out, and supply, all the missing information, and state what position the Vipers are in the league. [4]

- (e) The award of 1 point for scoring 2 goals or more is a new feature this season. The original proposal, however, was that the point should be awarded for scoring 3 goals or more.

If the original proposal had been adopted, which teams would now be first, second and third, and with how many points? [3]