Cambridge International AS & A Level

THINKING SKILLS	**9694/33**
Paper 3 Problem Analysis and Solution	**May/June 2023**
	2 hours

You must answer on the enclosed answer booklet.

You will need: Answer booklet (enclosed)
                  Calculator

INSTRUCTIONS

- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- Show your working.
  Where a final answer is incorrect or missing, you may still be awarded marks for correct steps towards a solution.
  In most questions, full marks will be awarded for a correct answer without any working. In some questions, however, you will not be awarded full marks if working needed to support an answer is not shown.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

[Turn over

1 The scarves of the Rainbows baseball team consist of alternating black and 'rainbow' stripes along the whole length of the scarf. Each scarf is 1.6 metres long and has 6 black stripes and 5 rainbow stripes.

All of the black stripes are the same length as each other.
A rainbow stripe consists of 7 narrow stripes, one in each colour of the rainbow.
Each narrow stripe is 2 cm long, so a rainbow stripe is 14 cm long.

(a) Show that each black stripe is 15 cm long. [1]

The scarves are knitted with wool that comes in balls containing 20 metres of wool.
Whenever two pieces of wool need to be joined, this is achieved without wasting any wool.
Each 1 cm length of scarf requires 20 cm of wool.

(b) How many balls of red wool are needed to knit 24 scarves? Justify your answer. [2]

The team has bought 8 balls of wool in each rainbow colour and 75 balls of black wool.

(c) **(i)** Show that there is sufficient green wool to knit 80 Rainbow scarves. [1]

(ii) Is there sufficient black wool to complete these scarves? Justify your answer. [2]

The wool that the team has bought will be used to make as many scarves as possible.
The cost of the wool was $6 a ball or $20 for 4 balls of the same colour.

The scarves will be sold for a whole number of dollars each.

(d) What is the least amount for which a scarf could be sold so that the team covers the cost of buying the wool? [4]

[Turn over for Question 2]

2 Julie sells sweets in her shop. There are five different types of sweet available, each of which is a different colour. All sweets weigh a small whole number of grams. Sweets of the same colour do not necessarily weigh the same amount.

Customers put the sweets that they wish to buy in one or more bags. The price for a bag of sweets is $1.00 for the bag, plus an amount for every complete 100 g of sweets, which is determined by the most expensive type of sweet in the bag. The amounts per 100 g are shown in the table.

Sweet colour	Red	Yellow	Green	Blue	Purple
Price per 100 g	$0.30	$0.50	$0.70	$0.80	$1.00

Julie's first customer today buys 482 g of red sweets, 507 g of yellow sweets and 442 g of green sweets.

(a) Show that it costs $10.80 to buy these sweets if they are all placed in one bag. [2]

(b) How much would it cost to buy the sweets if they were bought in three bags with just one colour of sweet in each bag? [2]

The customer in fact puts the sweets into bags in such a way that she pays the least possible total cost for the sweets.

(c) What is this least possible total cost? [3]

Julie's second customer today wants to buy some yellow sweets and purple sweets and has $14.00 to spend.

(d) What is the maximum possible total weight of the sweets bought if the customer buys as many sweets as possible with

   (i) equal weights of yellow and purple sweets? [2]

   (ii) **exactly** twice the weight of purple sweets as yellow sweets? [3]

Julie has decided to change the way in which the price of a bag of sweets is calculated. There will no longer be a charge for the bag, but at least 500 g of sweets must be placed in any bag bought. The price for blue sweets will now be $0.95 for every complete 100 g.

(e) What is the least weight of blue sweets that will be more expensive with this new system compared with the old one? [2]

The price of purple sweets will be set so that bags of purple sweets are always more expensive with this new system compared to the old one.

(f) What is the lowest value that could be set for the price for every complete 100 g of purple sweets? [1]

3 Multi-Facto is a game in which two players take turns in taking numbers from a list, with the goal of being the person who takes the last number. The list consists of all the numbers from 1 up to a maximum that has been agreed by the players. Once a number has been taken from the list, it cannot be used again by either player.

A turn involves selecting a string of numbers, each of which must be a **factor** or a **multiple** of the previous number in the string. A string may consist of only one number, but the player must continue their string until there are no continuations possible.

(A *factor* of a number is a whole number that divides into it exactly (including 1).)
(A *multiple* of a number is the result of multiplying it by a whole number.)

Below is an example of a string of five numbers that a player could take at the start of a game with a list of 20.

$$16 \rightarrow 4 \rightarrow 1 \rightarrow 14 \rightarrow 7$$

Luciano and Jenny are playing a game with a list of 12. Luciano is to take the first turn. He is considering what lengths of string he could make.

(a) Give the only two possible strings of two numbers that could be the first turn. [1]

(b) How many different ways could Luciano start the game with a string of three numbers? Explain your answer. [3]

Luciano correctly believes that he can win the game by creating a string of ten numbers that starts with a 5. The remaining two numbers will not make a string, so Jenny will take one, and he will take the final one, thereby winning the game.

(c) Give a possible string of ten numbers from a list of 12. [2]

Luciano wins their first game with this tactic. After the game, Jenny considers whether she could create a string which used every number in the list.

(d) (i) Explain why this is not possible with a list of 12. [1]

   (ii) Show how it can be done with a list of 6. [1]

They play their next game with a list of 20. After two turns, only the following numbers are left:

$$2 \quad 4 \quad 6 \quad 8 \quad 11 \quad 12 \quad 13 \quad 16 \quad 17 \quad 18 \quad 19$$

It is Jenny's turn, and she believes that she can select a string on this turn that will ensure that ultimately she wins the game.

(e) Explain how she can do this. [2]

4 The Bolandian Environment Agency is planning to plant trees on plots of land formerly used for industry. All the plots are rectangular (or square). There are strict regulations on which types of trees must be planted on these plots, and how they must be planted.

Within the restrictions, as many trees as possible must be planted.

Pine trees must be planted in straight rows with exactly 2 m between individual trees in a row and with exactly 2 m between rows of trees. The rows must be parallel to a boundary of the plot. There must be a gap of at least 2.5 m between any pine tree and the boundary of the plot of land.

Wilfred works for the environment agency and he has been put in charge of planting the trees for a plot measuring 25 m by 25 m.

(a) Show that there would be 11 pine trees in each row. [1]

The boundaries must be planted with beech trees 0.5 m apart.

The costs of trees are shown in the table.

	1 tree	Batch of 5 trees	Batch of 25 trees	Batch of 100 trees
Pine	$20	$95	$460	$1800
Beech	$10	$45	$200	$700

Environment agency rules say that employees must not buy more trees than are needed for each plot, and they must pay the lowest possible price for the trees that they buy.

**(b)** What is the total cost of all the trees needed for Wilfred's plot? [3]

Sookie also works for the environment agency and she has been put in charge of planting the trees for a plot measuring 40 m by 35 m.

(c) (i) What is the total cost of all the trees needed for Sookie's plot? [3]

(ii) How much would be saved if the trees needed for Wilfred's and Sookie's plots were bought together? [2]

Wilfred tells his supervisor that he and Sookie will not be able to buy all the pine trees required by the regulations for their plots, because they have been given a combined budget for pine trees of only $5000. The supervisor tells Wilfred that they should buy as many pine trees as they can, and he should plant up to a third of these on his plot; but he must have the same number of trees in each row.

(d) What is the greatest number of pine trees that Wilfred can plant in his plot? [2]

The supervisor decides to increase the budget so that all the trees required by the regulations can be bought. She also now notices on the plans that Wilfred's and Sookie's plots are next to each other, and decides to treat them as one single plot.
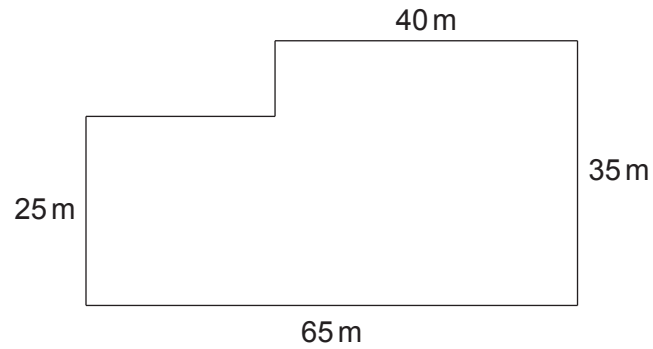
All the trees for this plot will be bought together. Wilfred calculates the saving made on the cost of trees, compared with what the cost would have been if he and Sookie had bought the trees for their two separate plots together.

(e) How much is this saving? [4]

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