ADDITIONAL MATHEMATICS

Paper 0606/01

Paper 1

General comments

The overall standard of work was very pleasing and there were many excellent scripts. Only a small percentage of candidates were unable to show what they had achieved, and most of these should not have been entered for this subject. Presentation was generally good, but there are still Centres who allow candidates to split the page into two halves and work on both halves – a practice which makes marking very difficult.

Comments on specific questions

Question 1

Although this was a very basic question on set notation, it presented most candidates with considerable difficulty. Indeed there were many very good candidates who only lost marks on this particular question. In part **(i)**, the use of the symbol "$\notin$" for 'is not an element of', was not widely appreciated. In part **(ii)** 'n(B) ≠ 16' was a common error, and in part **(iii)** such incorrect statements as '$C \cap D = 0$' or 'n($C \cap D$) = $\varnothing$' were very common.

Answer: **(i)** $x \notin A$; **(ii)** $n(B') = 16$; **(iii)** $C \cap D = \varnothing$ or $n(C \cap D) = 0$.

Question 2

Responses varied considerably from Centre to Centre. Many candidates had no difficulty in writing down the correct values of a, b and c. Other candidates struggled to find these values by attempting to use simultaneous equations rather than appreciating that a was the amplitude, that b was the number of full oscillations in 360º and that c was the intercept on the y-axis.

Answer: **(i)** 2; **(ii)** 3; **(iii)** –1.

Question 3

This was very well done by nearly all candidates. The majority realised the need to use function of a function and only a small proportion failed to multiply by 3. Candidates using the quotient rule were also successful, though the differential of '8' often appeared as '1' instead of 0. Answers to part **(ii)** were accurate with the majority realising that the change in y was the answer to part **(i)** multiplied by p.

Answer: **(i)** −6; **(ii)** −6p.

Question 4

Again responses varied considerably from Centre to Centre, with a good proportion of candidates obtaining full marks. A large number of candidates however failed to realise the need to find the magnitude of $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ and then multiply $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ by 2 and 3 respectively. The answers $\overrightarrow{OP} = 10(3\mathbf{i} - 4\mathbf{j})$ and $\overrightarrow{OQ} = 15(4\mathbf{i} + 3\mathbf{j})$ were common errors. In part **(ii)**, most candidates realised that $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$, though $\overrightarrow{PQ} = \overrightarrow{OQ} + \overrightarrow{OP}$ was often seen. Evaluation of λ by equating '$\lambda^2 \times 13$' to '$\left|\overrightarrow{PQ}\right|^2$' was well done.

Answer: **(i)** $6\mathbf{i} - 8\mathbf{j}$, $12\mathbf{i} + 9\mathbf{j}$; **(ii)** 5.

Question 5

Candidates coped better with this question than those in previous years. Most had little difficulty in writing down two appropriate matrices in part **(i)**. Unfortunately in part **(ii)**, the two matrices used were not compatible and it was very common to see the product of a 3×4 matrix and a 4×1 matrix appearing as a 1×3, instead of a 3×1 matrix. These same candidates however often recovered in parts **(iii)** and **(iv)** by correctly rewriting the 3×1 matrix as a 1×3 matrix. The majority of candidates obtained a correct final answer.

Answer: **(i)** $\begin{pmatrix} 5 & 8 & 4 & 10 \end{pmatrix} \begin{pmatrix} 300 & 60 & 40 \\ 150 & 50 & 20 \\ 120 & 40 & 0 \\ 100 & 0 & 0 \end{pmatrix}$; **(ii)** $\begin{pmatrix} 4180 & 860 & 360 \end{pmatrix}$; **(iii)** $\begin{pmatrix} 0.05 \\ 0.10 \\ 0.20 \end{pmatrix}$; **(iv)** 367.

Question 6

This was extremely well answered with a high proportion of candidates obtaining full marks. Working was clear and it was rare to see the binomial coefficients missing. Occasionally however ${}^6C_1$ came as ${}^6C_2$ and ${}^6C_2$ as ${}^6C_3$. Powers of 2 and $-\frac{1}{2}x$ were also correctly used, though errors occurred by writing $\left(-\frac{1}{2}x\right)^2$ as $-\frac{1}{4}x^2$ or as $\frac{1}{2}x^2$. Most candidates realised the need to consider two terms in x to find the value of k.

Answer: 3.

Question 7

Again, solutions varied considerably. A minority of candidates realised that because f was a quadratic function, the range was not automatically 53 to 89. Surprisingly, a significant minority of candidates offering this incorrect answer went on to obtain a correct minimum value of −11 in part **(ii)**. The x-value for the turning point usually came from calculus, with only a small minority of candidates realising that the minimum value occurred at the x-value obtained by setting $(x - \frac{1}{3})$ to 0. Solutions to part **(ii)b** varied considerably with only the more able candidates realising that if $y = f(x)$ has a minimum at $(\frac{1}{3}, -11)$, then $y = |f(x)|$ has a maximum at $(\frac{1}{3}, 11)$.

Answer: **(i)** $-11 \varnothing f(x) \varnothing 89$; **(ii)(a)** $(\frac{1}{3}, -11)$, minimum, **(b)** $(\frac{1}{3}, 11)$, maximum.

Question 8

Part **(a)** was generally well done, though the usual errors of writing lg $(x + 12)$ as lg x + lg 12 or failing to recognise that 1 = lg10 occurred with weaker candidates. The equation $x + 12 = 10(2 - x)$ was accurately solved. Part **(b)** was also reasonably answered, especially by candidates who converted $\log_2 p = a$ to $p = 2^a$ and $\log_8 q = b$ to $q = 8^b$. It was common for these candidates to proceed directly to $c = a - 3b$, though $c = \frac{a}{3b}$ was a frequent error. Those candidates who started by converting $\frac{p}{q} = 2^c$ to logarithmic form experienced problems through failure to convert from $\log_2 q$ to $\log_8 q$.

Answer: **(a)** $\frac{8}{11}$; **(b)**1$c = a - 3b$.

Question 9

The differentiation required in part **(i)** was generally well done, usually by the quotient rule. The algebraic error of expressing $-(2x - 4)$ as $-2x - 4$ was very common. The explanation as to why the curve had no turning points varied considerably, with many candidates unable to state such answers as 'there are no solutions to the equation $\frac{dy}{dx} = 0$' or '$\frac{dy}{dx}$ is always positive'. In part **(ii)**, a large number of candidates misread the question and took Q to be the point where the curve, rather than the tangent, crossed the y-axis. Calculating the coordinates of P and using a correct method for the area of the triangle was nearly always correct.

Answer: **(i)** $\frac{10}{(x+3)^2}$; **(ii)** 0.8.

Question 10

Part **(i)** proved to be too difficult for many candidates. Only about a half of all attempts realised that the cubic could be expressed as $(x-1)(x-k)(x-k^2)$ and that f(2) = 7. The majority of these expanded the brackets first and then substituted $x = 2$. Only a small proportion of candidates realised that the solution was much quicker if $x = 2$ was used before the brackets were expanded. Part **(ii)** was much more accessible and was well answered. Most candidates obtained $k = 3$ by trial and improvement and proceeded, either by inspection or division, to the correct quadratic $k^2 + k + 1$. Only about a half of candidates obtaining this quadratic realised and demonstrated that because '$b^2 - 4ac$' was negative, the cubic equation could have only one root.

Answer: **(ii)** $k = 3$.

Question 11

Part **(a)** was well answered. Most candidates used the identity $\cot x = \frac{1}{\tan x}$ to arrive at the correct quadratic equation for tan x. Most proceeded to state that tan x = 1 or −2 and obtained 4 correct answers in the required range. A few surprisingly stated that 'tan x = −2' had no solutions and others stated that if 'tan x (tan x + 1) = 2' then 'tan x = 2 or tan x + 1 = 2'.

In part **(b)**, most candidates stated that sin $(2y + 1) = -\frac{5}{6}$, though weaker candidates often expanded sin $(2y + 1)$ as sin $2y$ + sin 1. Errors caused by mixing degrees and radians were common as was the error in finding quadrants for $2y$, rather than for $2y + 1$. Careless premature approximation also led to the loss of accuracy marks.

Answer: **(a)** 45º, 116.6º, 225º, 296.6º; **(b)** 1.56 radians, 2.15 radians.

Question 12 EITHER

This was the more popular of the two alternatives, and the question generally proved to be a source of high marks. Part **(i)** was well answered, though (0, 4) was a common error. In part **(ii)** most candidates realised the need to differentiate, though the incorrect answers of e^{-2x} and $2xe^{-2x}$ were common. Several candidates misinterpreted 'normal' as 'tangent' and many others, in attempting to obtain the equation of the line, left the gradient as an algebraic expression, thereby obtaining a non-linear equation. In part **(iii)**, most candidates realised the need to evaluate the two areas separately. Most obtained the area of the triangle by using '$\frac{1}{2}bh$', though a few integrated the equation of the normal between 0 and 6. Integration of $4 - e^{-2x}$ was reasonable, though often $\int 4\,dx$ was taken to be 0, and $\int e^{-2x}\,dx$ was left as e^{-2x} or as $-e^{-2x}$.

Answer: **(i)** (0, 3), (–0.693, 0); **(ii)** (6, 0).

Question 12 OR

Candidates attempting this question generally scored well on the first few parts. The plotting of lg y against *x* was accurate and most realised that the values of *A* and *b* could be obtained from the intercept on the *y*-axis and the gradient respectively. Unfortunately, many candidates failed to cope with the sign of *A* and finished with $A = -2$, whilst many others took the gradient as *b* instead of lg *b*. Part **(iii)** was well answered with most candidates obtaining the value of *x* by setting lg *y* to 1. Part **(iv)** proved difficult, and although candidates could transform the equation $y^5 = 10^{-x}$ to $\lg y = -\frac{x}{5}$, very few were able to draw the line '$\lg y = -\frac{1}{5}x$' on the graph and hence find the value of *x* at the point of intersection of the two lines.

Answer: **(ii)** $A = 2.0$ (±.05), $b = 1.2$ (± 0.02); **(iii)** 38 (± 0.5); **(iv)** 7 (± 0.5).

ADDITIONAL MATHEMATICS

Paper 0606/02

Paper 2

General comments

Although some questions inevitably produced a greater percentage of correct responses, each of the questions was answered correctly by a large number of candidates and none of the questions appeared to be excessively difficult. One of the most common causes of lost marks was carelessness in simple manipulative algebra, as in the following few examples:

$y - 4x + 8 = 0$ written as $y = 4x + 8$,

$-\frac{24+8\sqrt{5}}{4}$ simplified to $-6+2\sqrt{5}$.

Comments on specific questions

Question 1

This question proved to be very easy for able candidates who simply wrote the answers down. Weak candidates did not understand what was required; in part **(i)**, for example, some thought the answer was $f(x) + 2$, whilst others offered $f^{-1}(x) = (x-2)^{\frac{1}{3}}$, corresponding to $f(x) = x^3 + 2$. Some candidates showed a misunderstanding of the way in which the function symbols should be combined, giving their answers as fg, fg^{-1} and gf^{-1}.

Answers: **(i)** gf; **(ii)** $g^{-1}f$; **(iii)** $f^{-1}g$.

Question 2

Only the weakest candidates failed to obtain full marks. The usual method of solution was to replace cot *x* by $\frac{\cos x}{\sin x}$ and then combine the terms on the left-hand side, although some candidates achieved the desired result by replacing cos *x* and sin *x* by $\frac{\cot x}{\operatorname{cosec} x}$ and $\frac{1}{\operatorname{cosec} x}$ respectively. Careless notation was occasionally seen, e.g. $\frac{\cos}{\sin}x$.

Question 3

The only common error made by better candidates was on integration when $\cos(2x + \frac{\pi}{6})$ was multiplied by $\frac{1}{2}$, 2 or – 2 rather than $-\frac{1}{2}$. Weaker candidates produced a variety of errors including $\frac{1}{2}\cos(2x + \frac{\pi}{6})^2$, $\cos(x^2 + \frac{\pi x}{6})$ and $(x^2 + \frac{\pi x}{6})\cos(x^2 + \frac{\pi x}{6})$. Some first took $\sin(2x + \frac{\pi}{6})$ to be $\sin 2x + \sin \frac{\pi}{6}$ whilst others abandoned the brackets so that the integral contained $\cos 2x + \frac{\pi}{6}$ which were then treated as two separate terms. Although virtually all candidates understood how to use the limits, far too many used the incorrect mode on their calculators, obtaining values corresponding to $\frac{\pi}{6}$ degrees. The exact answer, $\frac{\sqrt{3}}{4}$, was rarely seen and most correct answers were of the form 0.433, although many of these then became 0.43, presumably because of the candidates' incorrect understanding of the phrase '3 significant figures'.

Answer: $\frac{\sqrt{3}}{4}$ or 0.433.

Question 4

Although providing little difficulty for the best candidates, this question probably produced the least number of correct responses. Many candidates did not understand the implication of the phrase 'in still water' and so gave the resultant speed, 1.5 ms^{-1}, as the answer to part **(i)**; this was occasionally followed by an answer of 90° to part **(ii).** Others failed to convert 1 minute to 60 seconds and so found the resultant speed to be 90 ms^{-1}. Most candidates appreciated that a triangle - usually of velocities, but occasionally of displacements - was required, but quite a number of these indicated that the ferry was headed downstream rather than upstream. A vector equation was sometimes used to indicate the additive relationship between the sides of the vector triangle, but in virtually every case the vectors were replaced by scalar quantities so that the resultant velocity became 3.5 or 0.5 from 2 $\pm$ 1.5. Some right-angled triangles were nonsensical with, for instance, a smaller side of 2 and a hypotenuse of 1.5. In part **(ii)** very many candidates failed to answer the question which was asked, namely, to find 'the angle to the bank'; thus 53.1°, from $\tan^{-1}\left(\frac{2}{1.5}\right)$, was an extremely common answer. Some candidates assumed that *AB* was due north and gave the answer as a bearing, 306.9°.

Answers: **(i)** 2.5 ms^{-1}; **(ii)** 36.9°.

Question 5

This question resulted in a very large number of completely correct solutions. Most candidates substituted $14 - 2x$ for y and subsequent failure to obtain the correct quadratic in x was invariably due to carelessness, e.g. writing $2x^2 - (14 - 2x)^2 \equiv 2x^2 - 196 - 56x + 4x^2$. A few candidates substituted $\frac{14-y}{2}$ for x but some then obtained $196 - 28y + y^2$ from $2x^2$ either through regarding $2x^2$ as $(2x)^2$ or by taking x^2 to be $\frac{(14-y)^2}{2}$. The ideas of factorisation and the distance between two points were known to all but the weakest candidates.

Question 6

Weak candidates did not appreciate that differentiation was necessary in part **(i)**. The coefficient of x, i.e. a, was frequently taken to be 3. Some candidates gained a little credit for obtaining, or implying, $7 = 2^3 + 2a + b$ but then assumed spurious values of x and y in order to evaluate a and b. Despite the simplicity of the differentiation various erroneous forms of $\frac{dy}{dx}$ were seen including, $3x^2 + a + b$, $3x + a$, $2x^2 + a$ and $2x + a$. Part **(ii)** was frequently answered incorrectly even by candidates who had obtained the correct values of a and b, e.g. writing $3x^2 - 9 = 3 \Rightarrow 3x^2 = 6$ and $x^2 = 4 \Rightarrow x = \pm 4$. Some obtained $x^2 = 4$ but could only find $x = 2$, $y = 7$ whilst others, who understood that there were two values of x, discarded $x = -2$ as 'not applicable'. Not every candidate who selected $x = -2$ could then obtain the correct value of y, e.g. $y = -8 - 18 + 17 = -9$. Some candidates misunderstood the question and attempted to find the point where the tangent intersected the curve.

Answers: **(i)** $a = -9$, $b = 17$; **(ii)** $(-2, 27)$.

Question 7

(a) Nearly all candidates obtained a little credit for equating the two expressions for y, but the weaker candidates could make no further progress, often assuming that $m = 1$. Most of the better candidates then rearranged the equation as a quadratic in x for which the discriminant was zero., thus obtaining the value of m; this value was usually substituted in the quadratic which was then solved to find x. The other, less frequent, approach was to replace m in the equation $mx - 3 = x + \frac{1}{x}$ by $\frac{d}{dx}(x + \frac{1}{x})$. Both approaches were used successfully but in each case some candidates overlooked the second demand of the question, i.e. having evaluated m they made no attempt to find x, and vice versa.

(b) Candidates found this 2-mark part question to be very difficult. Those candidates who began with $(x + 5)(x - 3) < 0$ usually had no difficulty in evaluating c and d, although a few found d to be -15. Many candidates attempted to solve simultaneous equations - those using $25 - 5c - d < 0$ and $9 + 3c - d < 0$ usually arrived at $c < 2$, $d < 15$. Because of the inequality signs some candidates attempted to solve equations, or inequalities, in which x took the values -4 and 2. Many weaker candidates did not attempt this part of the question.

Answers: **(a)** $m = -\frac{5}{4}$, $x = -\frac{2}{3}$; **(b)** $c = 2$, $d = 15$.

Question 8

The inverse matrix, $\mathbf{A}^{-1}$, was usually correct; the only common error lay in taking the determinant of $\mathbf{A}$ to be 11 rather than 5. In part **(i)** the weakest candidates merely solved the simultaneous equations by eliminating one of the variables. Many other candidates ignored the instruction to 'use the inverse matrix of **A**' and used a different inverse to deal with $\begin{pmatrix} 1 & -4 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} -8 \\ -1 \end{pmatrix}$, $\begin{pmatrix} -4 & 1 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} 4 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$, or simply multiplied $\begin{pmatrix} 8 \\ 1 \end{pmatrix}$ by $\mathbf{A}^{-1}$ - this was frequently indicated as post-multiplication by $\mathbf{A}^{-1}$ rather than pre-multiplication. In part **(ii)** the instruction to 'use the inverse matrix of **A**' was again ignored or overlooked with many candidates replacing **B** by $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and setting up a system of simultaneous equations. Some candidates thought that division of matrices was possible and divided each element of $\begin{pmatrix} -2 & 3 \\ 9 & -1 \end{pmatrix}$ by the corresponding elements of **A**. Most candidates attempting to multiply two (2 × 2) matrices understood the process, but some thought that each element of the product was obtained from the product of two corresponding elements of the two matrices. Again a considerable number of candidates failed to appreciate the significance of the order of multiplication and pre-multiplied, rather than post-multiplied, $\begin{pmatrix} -2 & 3 \\ 9 & -1 \end{pmatrix}$ by $\mathbf{A}^{-1}$.

Answers: **(i)** $x = 3$, $y = 4$; **(ii)** $\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$.

Question 9

(a) There were occasional errors in expanding $(2 - \sqrt{5})^2$ but most candidates were able to obtain $9 - 4\sqrt{5}$. Surprisingly, rather than simplifying the term $-\frac{8}{3-\sqrt{5}}$ by rationalising the denominator, a large majority of candidates preferred to first combine $9 - 4\sqrt{5}$ and $-\frac{8}{3-\sqrt{5}}$, a process which increased the possibility of error and sometimes resulted in $(9 - 4\sqrt{5})(3 - \sqrt{5}) - 8$, the denominator $3 - \sqrt{5}$ having disappeared. Arithmetic and sign errors were very common.

(b) For many weak candidates the only credit gained was for understanding that $(a^{y+1})^2$ became a^{2y+2}, although some failed to realise this, taking $(a^{y+1})^2$ to be a to the power $y^2 + 2y + 1$ or, occasionally, $2y + 1$.

Errors such as $\frac{a^x}{a^{2y+2}} = a^{x-2y+2}$ and $\frac{b^y}{b^{3-x}} = b^{y-3-x}$ were extremely common. Some candidates treated the left-hand side as though addition rather than multiplication was involved, finding a 'common denominator', i.e. $\frac{a^x}{b^{3-x}} \times \frac{b^y}{a^{2y+2}} \equiv \frac{a^x a^{2y+2} \times b^y b^{3-x}}{b^{3-x} a^{2y+2}}$. Powers were frequently combined incorrectly, e.g. $a^x \times b^y = ab^{x+y}$; ab^6 was sometimes treated as being $(ab)^6$ and the power of a in ab^6 was occasionally taken to be 0. Despite the many errors seen, strong candidates had little difficulty in obtaining full marks.

Answers: **(a)** $3 - 6\sqrt{5}$; **(b)** $x = 7$, $y = 2$.

Question 10

(a) Many candidates thought the answer was 126, obtained from ${}^9C_4$. The answer 362 880, from 9!, was also seen occasionally.

(b) The majority of candidates had little difficulty with this part of the question and it was answered correctly more often than part **(a)**. A few candidates misunderstood the question and thought 5 people were being chosen from 26, i.e. 13 entertainers, 8 singers and 5 comedians. Some candidates interchanged the operations of addition and multiplication, obtaining 4950 from $({}^5C_1 + {}^8C_4) \times ({}^5C_2 + {}^8C_3)$. A handful of candidates succeeded with the more complicated method of subtracting the number of ways of forming unacceptable combinations of comedians and singers from ${}^{13}C_5$.

Answers: **(a)** 3024; **(b)** 910.

Question 11

Strong candidates dealt easily with this question but it proved difficult for weaker candidates. Part **(i)** was soon abandoned by those candidates who did not perceive that the differentiation of a product was involved. Differentiation of $e^{-\frac{x}{2}}$ was not always correct, the most common mistake being $-\frac{1}{2}xe^{-\frac{x}{2}}$. Strangely, candidates who negotiated part **(i)** correctly often failed to recognise that, in part **(ii)**, once again the differentiation of a product was required. Candidates frequently differentiated correctly but then made errors in attempting to combine and simplify the terms resulting from differentiation. Those candidates attempting part **(iii)** almost always understood that the solution of $\frac{dy}{dx} = 0$ was required; those failing to do so usually attempted to solve $\frac{d^2y}{dx^2} = 0$ or $y = 0$. The obvious conclusion from $\frac{dy}{dx} = 0$ that $2 - x$ must be zero was frequently overlooked and there were many spurious attempts, usually involving logarithms, to solve $2e^{-\frac{x}{2}} = xe^{-\frac{x}{2}}$ or $e^{-\frac{x}{2}} = 0$, with log 0 being taken as 1 or 0. In part **(iv)** interpretation of the sign of $\frac{d^2y}{dx^2}$ was almost always correct, although the evidence which led to this interpretation was often faulty, in that expressions for $\frac{d^2y}{dx^2}$ were frequently incorrect. Some candidates offered a one-word answer - usually 'maximum' - without any explanation or evidence, and 'maximum' was also the usual response from those candidates who found the relevant value of $\frac{d^2y}{dx^2}$ to be zero. Consideration of the signs of $\frac{dy}{dx}$ was rarely used.

Answers: **(ii)** $\frac{1}{4}(x-4)e^{-\frac{x}{2}}$; **(iii)** $x = 2$, $y = \frac{2}{e}$; **(iv)** Maximum.

Question 12 EITHER

This alternative was the less popular of the two and resulted in a smaller proportion of correct answers with part **(i)** being the stumbling block for many candidates. All understood that the length of arc *LM* was $r\theta$, so that the crux of the problem lay in expressing two of the sides of a right-angled triangle in terms of the remaining side and one of the acute angles. Unfortunately, many candidates overcomplicated this task and obtained expressions which, although correct, were often difficult to handle and induced later errors. Some, having obtained $LN = r\tan\theta$, found *ON* to be $\sqrt{r^2 + r^2\tan^2\theta}$, which became, for weaker candidates, $r + r\tan\theta$. Others realised that $\cos\theta = \frac{r}{ON}$ but then manipulated this to $ON = \frac{\cos\theta}{r}$. Some found it necessary to use the sine rule, so that *LN* became $\frac{r\sin\theta}{\sin(\frac{\pi}{2}-\theta)}$, leading to $ON = \sqrt{r^2 + \frac{r^2\sin^2\theta}{\sin^2(\frac{\pi}{2}-\theta)}}$. Although it was not necessary to express *ON* as $r\sec\theta$, it was noticeable how rarely this notation was seen, with $\frac{r}{\cos\theta}$ much preferred - this sometimes led to the unnecessary step of bringing terms to a common denominator of $\cos\theta$. A few weak candidates thought that *P* could be obtained by subtracting the perimeter of the sector *OLM* from the perimeter of the triangle *OLN*, whilst others assumed that *M* was the mid-point of *ON*, leading to $LN = \sqrt{(2r)^2 - r^2}$ and $P = \sqrt{3}r + r + r\theta$. Candidates were much more successful in dealing with part **(ii)** and the expression for *A* was often correct. In attempting to evaluate *r* and *A* not all candidates were aware that the trigonometric functions involved 1.2 radians rather than 1.2 degrees.

Answers: **(i)** $r\theta + r\tan\theta + r\sec\theta - r$; **(ii)** $\frac{1}{2}r^2\tan\theta - ½r^2\theta$; **(iii)** 15.0; **(iv)** 154.

Question 12 OR

Although many of the candidates attempting this question obtained full marks, few seemed able, or willing, to use the most direct lines of reasoning. This comment particularly applies to part **(i)**, where it often seemed that candidates needed to justify their answers with complicated work. Thus the *x*-coordinate of *C***,** which was obviously $\frac{1}{2}(6 + 3)$, was most often reached by equating the lengths of *AC* and *BC*. Similarly the *y*- coordinate of *C*, which was clearly 4 less than the *y*-coordinates of *A* and *B* (since *AB* = 3 units and the area of triangle *ABC* = 6 square units), was often attempted via the array method for area - a method which some weak candidates failed to apply correctly, either omitting the $\frac{1}{2}$ or using an array with only 3 columns. The method worked well for able candidates who understood that ± 6 should be considered and resolved the ambiguity of sine correctly. Weak candidates simply used 6 and so were dependent on whether they inserted their coordinate into the array in a counter-clockwise or a clockwise fashion. Those who found the *y*-coordinate of *C* to be +7 almost always continued to use this figure, despite the diagram clearly indicating that the value must be negative. The various methods employed to deal with parts **(ii)**, **(iii)** and **(iv)** were almost always correct in principle. In part **(ii)** vectors or the mid-point idea were used, while in part **(iii)** $x = 10$ was substituted in the equation of *DE*, m_{DE} was equated to m_{AC} or the vector *DE* was taken to be $\mu \times$ the vector *AC*. In part **(iv)** the approaches used were almost exclusively based either on Pythagoras, showing $DE^2 + EC^2 \neq CD^2$, or on perpendicular lines, showing $m_{EC} \times m_{ED} \neq -1$; it was refreshing to see an unusual approach from a candidate who understood that if $\angle CED = 90°$, then *BE* = *BC* and this was not the case.

Answers: **(i)** $(4\frac{1}{2}, -1)$; **(ii)** $(7\frac{1}{2}, 7)$; **(iii)** $\frac{1}{3}$.