Cambridge IGCSE
Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

1952650704

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/61**

Paper 6 (Extended) **October/November 2019**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** (questions 1 to 3) and **B** (questions 4 to 5).
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

This document consists of **12** printed pages.

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Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 to 3)

DECIMAL FORMS (20 marks)

You are advised to spend no more than 45 minutes on this part.

This investigation looks at the patterns when changing a fraction to its decimal form.

Examples

$\frac{2}{3} = 0.666\ldots = 0.\dot{6}$ This is a repeating decimal.

$\frac{3}{4} = 0.75$ This is a terminating decimal.

The fraction $\frac{5}{8}$ has a numerator of 5 and a denominator of 8.

1 This question is about terminating decimals.

(a) **(i)** Complete the table.

Fraction	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{7}{20}$	$\frac{1}{25}$	$\frac{3}{500}$
Equivalent fraction	$\frac{}{10}$	$\frac{}{10}$	$\frac{}{100}$	$\frac{}{100}$	$\frac{}{1000}$
Decimal	0.5	0.2			

(ii) What is always true about the denominators of equivalent fractions when the decimal form is a terminating decimal?

....

....

(b) **(i)** Write each number as a product of its prime factors.
The first two have been completed for you.

$20 = 2 \times 2 \times 5$

$25 = 5 \times 5$

$50 =$

$100 =$

$500 =$

(ii) Use your answers to **part (i)** to help you complete the table.

Fraction	Decimal	Number of decimal places	Denominator written as a product of primes using powers	Larger power
$\frac{1}{20}$	0.05	2	$2^2 \times 5$	2
$\frac{7}{25}$	0.28	2	5^2	2
$\frac{9}{50}$	0.18	2		
$\frac{19}{100}$	0.19			2
$\frac{13}{200}$	0.065	3	$2^3 \times 5^2$	3
$\frac{11}{500}$	0.022			
$\frac{17}{5000}$	0.0034		$2^3 \times 5^4$	4

(iii) A fraction with denominator $2^p \times 5^q$, where q is greater than p, is changed to its decimal form.

Write down the number of decimal places in the decimal form of this fraction.

....

2 This question is about repeating decimals.
The number of digits in the repeating pattern is called the *repeat length*.

Example

$\frac{1}{13} = 0.\underbrace{076923}\,076923\,076923\ldots = 0.\dot{0}76\,92\dot{3}$ This is a repeating decimal with a repeat length of 6.

(a) **(i)** Complete the table.

Fraction	$\frac{1}{3}$	$\frac{1}{11}$	$\frac{1}{37}$	$\frac{1}{111}$	$\frac{1}{41}$	$\frac{1}{7}$
Equivalent fraction	$\frac{3}{9}$	$\frac{9}{99}$	$\frac{}{999}$		$\frac{}{99\,999}$	$\frac{}{999\,999}$
Denominator of equivalent fraction	$10^1 - 1$	$10^2 - 1$		$10^3 - 1$		
Decimal	$0.\dot{3}$	$0.\dot{0}\dot{9}$	$0.\dot{0}2\dot{7}$			$0.\dot{1}42\,85\dot{7}$
Repeat length	1	2			5	6

(ii) A repeating decimal has a repeat length of *k*.

Write down an expression, in terms of *k*, for the denominator of this fraction.

....

(b) **(i)** $\frac{1}{407} = \frac{1}{11 \times 37} = \frac{1}{11} \times \frac{1}{37}$

$\frac{1}{407}$ is changed to its decimal form.

Show that this has a repeat length that is equal to the lowest common multiple (LCM) of the repeat lengths of the decimal forms of $\frac{1}{11}$ and $\frac{1}{37}$.

(ii) Show how the lowest common multiple (LCM) of the repeat lengths of $\frac{1}{7}$ and $\frac{1}{37}$ gives the repeat length of $\frac{1}{259}$.

(iii) m and n are different prime numbers.

The decimal form of $\frac{1}{m}$ has a repeat length of 6.

The decimal form of $\frac{1}{n}$ has a repeat length of 9.

Find the repeat length of the decimal form of $\frac{1}{m \times n}$.

....

3 Some decimals have non-repeating decimal parts followed by repeating decimal parts.

Example
$0.6\dot{5} = 0.65555\ldots$ In this decimal, the 6 does not repeat but the 5 does.

(a) Show that adding the decimal forms of $\frac{1}{5}$ and $\frac{1}{3}$ gives a decimal of this type.

(b) Complete the table.

Fraction	Decimal	Number of non-repeating decimal places	Repeat length	Denominator written as a product of primes using powers
$\frac{1}{6}$	$0.1\dot{6}$	1	1	2×3
$\frac{1}{12}$	$0.08\dot{3}$	2	1	
$\frac{7}{75}$				
$\frac{11}{24}$		3		
$\frac{317}{600}$	$0.528\dot{3}$			$2^3 \times 5^2 \times 3$
$\frac{1}{1320}$	$0.000\,\dot{7}\dot{5}$	3	2	$2^3 \times 5 \quad \times 11 \times 3$
$\frac{50001}{101750}$	$0.491\,\dot{4}10\,31\dot{9}$	3	6	$2 \times 5^3 \times 11 \times 37$

(c) A fraction is of the form

$$\frac{1}{2^a \times 5^b \times c \times d}.$$

In the fraction a and b are positive integers and c and d are different prime numbers less than 90.

The decimal form of this fraction has 5 non-repeating decimal places and a repeat length of 30.

Using **question 1(b)** and **question 2(a)(i)**, find a possible value for each of a, b, c and d.

a = ... b = ... c = ... d = ...

(d) m and n are different prime numbers.

The decimal form of the fraction $\frac{1}{m}$ has a repeat length of q.

The decimal form of the fraction $\frac{1}{n}$ has a repeat length of $3q$.

The decimal form of the fraction $\frac{1}{w}$ has

- k non-repeating decimal places

and

- a repeat length of $3q$.

Find a possible expression for w, in terms of k, m and n.

....

B MODELLING (QUESTIONS 4 to 5)

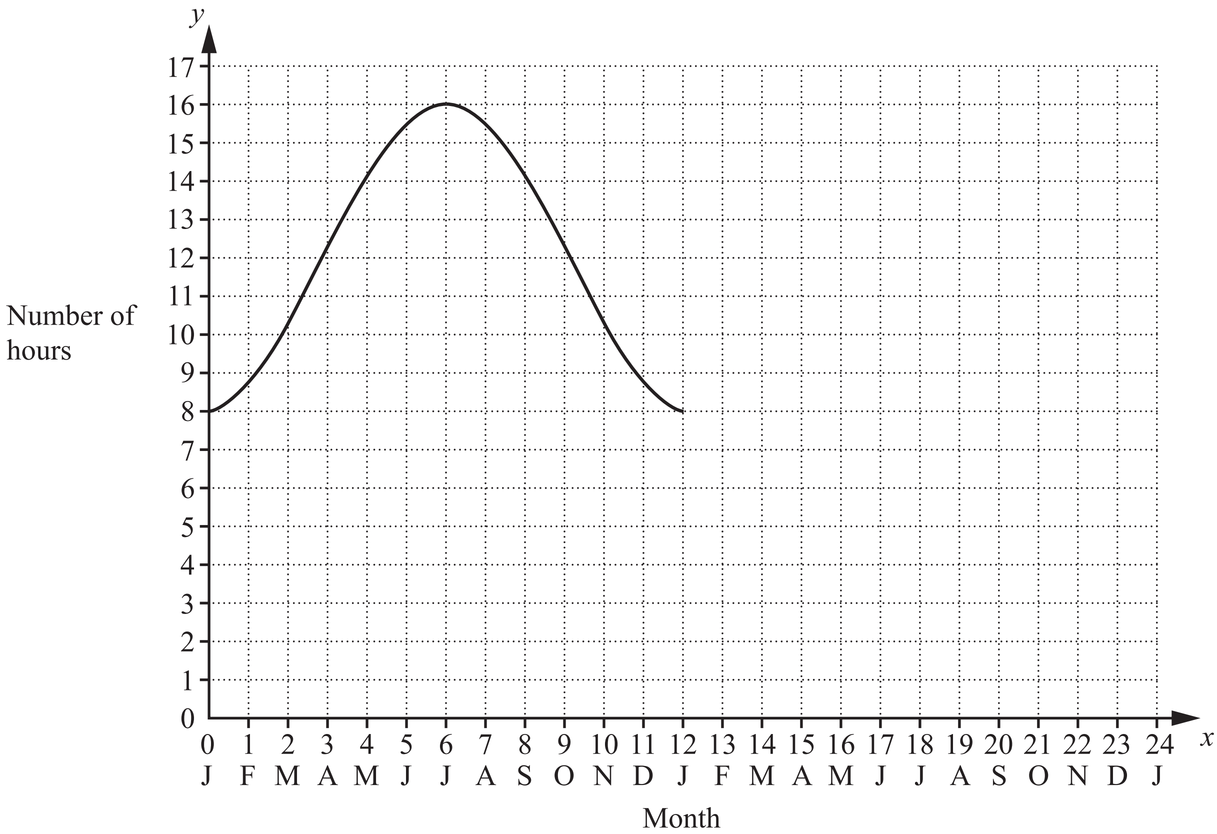
FLOWERING TIMES (20 marks)

You are advised to spend no more than 45 minutes on this part.

This task is about when plants flower.
The number of hours of darkness affects when plants flower.
In this investigation,

the number of hours of darkness + the number of hours of daylight = 24.

4 **(a)** The graph shows the approximate number of hours of daylight in Normandy, France for 2017.
On the x-axis, 0 is 1st January 2017, 12 is 1st January 2018 and 24 is 1st January 2019.



(i) The pattern for the number of hours of daylight remains the same each year.
Complete the graph to show the approximate number of hours of daylight for 2018.

(ii) On the same grid, draw the graph to show the number of hours of **darkness** for the two years.

(iii) Describe fully the **single** transformation that maps the graph of the number of hours of daylight onto the graph of the number of hours of darkness.

....

(b) Pierre grows oats in Normandy, France.
Oat plants flower when there are less than 12 hours of darkness.

Find the earliest month when an oat plant flowers.

....

(c) Pierre models the number of hours of **daylight**, p, using

$$p = 12 + 3.9\sin\left(30\left(x - 2\frac{20}{31}\right)\right)^{\circ}$$

where x has the following value at the start of each month.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
x	0	1	2	3	4	5	6	7	8	9	10	11

(i) Using Pierre's model, find the maximum number of hours of daylight and the month in which it occurs.

Maximum hours of daylight ....

Month ....

(ii) Pierre thinks that his oat plants flower at their best when the number of hours of **darkness** is at its minimum.

Find the minimum number of hours of **darkness**.

....

(iii) Using Pierre's model, write down a model for the number of hours of **darkness**, q.

....

(d) On 20th March, the number of hours of darkness is the same as the number of hours of daylight.
There are 31 days in March.

Show that Pierre's model finds this date accurately.

5 Alexa grows soybeans in Queensland, Australia.
In Australia, there are more hours of darkness in June than there are in January.

Alexa records the number of hours of darkness for 360 days, from 1st January to 26th December.
She finds this information.

Hours of darkness	Maximum	13.8
	Minimum	10.2

Alexa models the number of hours of darkness, y, by drawing the graph of

$$y = a - b\cos t°$$

where t is the day of the year.

(a) **(i)** Write suitable calculations to show that $a = 12$ and $b = 1.8$.

(ii) On the axes, sketch the graph of the model $y = 12 - 1.8\cos t°$ for $0 \leqslant t \leqslant 360$.

(b) Soybeans flower when there are more than 12 hours of darkness.
The flowers grow at the fastest rate when there are 13.6 or more hours of darkness.

(i) Find the number of days when the flowers are growing at their fastest rate.

....

(ii) The table shows the value of t on the first day of each month.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
t	1	32	60	91	121	152	182	213	244	274	305	335

On average, Alexa's soybeans flower 53 days after she plants them.
She wants to plant them so that, when they begin to flower, the flowers grow at the fastest rate.

Use Alexa's model to show that the latest date she should plant her soybeans is 3rd June.

Question 5(c) is printed on the next page.

(c) **(i)** Alexa uses her model, $y = 12 - 1.8\cos t°$, to find the first date when the number of hours of darkness is the same as the number of hours of daylight.

Find this date.

....

(ii) The actual date when the number of hours of darkness was the same as the number of hours of daylight was 20th March.

Alexa decides to change her model so that it finds this date accurately.
She does this by a translation of the graph of her model.

Find the new model.

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